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ABSTRACT

This self-study book contains 25 lessons in practical mathematics. The lessons include: (1) multiplication and division of whole numbers; (2) the four basic operations with fractions and decimals; (3) understanding mathematics symbols; (4) using formulae and solving and checking simple equations; (5) percentage problems and applications; (6) measurement of length, area, volume using English units of measure; (7) rates and averages; and (8) ratio and proportions. Course description, directions for testing and training, quizzes and answers are bound in a separate book (book I). (JBW)

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PRACTICAL ARITHMETIC SELF STUDY

R. E. Main

BOOK II--COURSE LESSONS

SE 019 852

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THE PRACTICAL ARITHMETIC SELF-STUDY (PASS) COURSE

BOOK II--COURSE LESSONS

R. E. Main

September 1973

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Navy Personnel Research and Development Center
San Diego, California 92152

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LESSON A

MULTIPLICATION

Multiplication: A Fast Way of Adding

If we have 6 boxes and each box holds 9 dress blue uniforms in it, then how many uniforms do we have in all?

Addition	Multiplication
9	9
+ 9	x 6
+ 9	54 uniforms
+ 9	
+ 9	
+ 9	
54 uniforms	

You can do it either way, but multiplication is faster.

Remembering the Multiplication Tables

Everyone forgets! I do, you do, we all do. Let's face it! Not many people use their math skills every day. What you don't use, you forget.

What can you do when you forget? Let's take an example. Suppose you can't remember what 7 x 9 is equal to. You could change the multiplication problem into an addition problem:

$$\boxed{7 \times 9 = \text{seven } 9\text{'s} = 9 + 9 + 9 + 9 + 9 + 9 + 9 = 63}$$

If you add the 7 nines together, you get ________.

Work these multiplication problems by addition.

3 x 8 = 3 eights and: 8 + 8 + 8 = ________

5 x 7 = 5 sevens and: __ + __ + __ + __ + __ = 35

Multiplying Larger Numbers

When you multiply, you start at the back end and work forward to the front end of the number.

Multiply 413 by 2.

1. First, we multiply the 3 by 2 and get 6

$$\begin{array}{r} 413 \\ \times\ 2 \\ \hline \underline{6} \end{array}$$

2. Next, we multiply the 1 by 2 and get 2

$$\begin{array}{r} 413 \\ \times\ 2 \\ \hline \underline{2}6 \end{array}$$

3. Finally, we multiply the 4 by 2 and get 8

$$\begin{array}{r} 413 \\ \times\ 2 \\ \hline \underline{8}26 \end{array}$$

How to Carry a Number

If you were to add: 28 + 7

You would do this:

$$\begin{array}{r} 1 \\ 28 \\ +\ 7 \\ \hline 35 \end{array}$$

How did we work the above problem? We had to carry a number. We added 8 and 7 to get 15. We put down the 5 and carried the 1. We added the 1 we carried to the 2 so our answer was _ 5.

We also carry numbers when we multiply. Here is an example:

Multiply 32 times 5

First you say 5 x 2 = 10
put down the 0 and carry the 1

$$\begin{array}{r} 32 \\ \text{x } 5 \\ \hline \underline{0} \end{array}$$

(carry the 1)

Next you say 5 x 3 = 15
plus 1 carried = 16
put down the 16

$$\begin{array}{r} 32 \\ \text{x } 5 \\ \hline \underline{1}60 \end{array}$$

Sometimes we have to carry into a zero. For example:

Multiply 508 times 3

First you say 3 x 8 = 24
put down the 4 and carry the 2

$$\begin{array}{r} 508 \\ \text{x } 3 \\ \hline \underline{4} \end{array}$$

(carry the 2)

Next you say 3 x 0 = 0
plus 2 carried = 2
put down the 2

$$\begin{array}{r} 508 \\ \text{x } 3 \\ \hline \underline{2}4 \end{array}$$

(no carry)

Finally you say 3 x 5 = 15
with nothing to carry
put down the 15

$$\begin{array}{r} 508 \\ \text{x } 3 \\ \hline \underline{1}524 \end{array}$$

Fill in the blanks.

Multiply 182 times 7

First you say 7 x 2 = ___
put down the __ and carry the 1

$$\begin{array}{r} 182 \\ \text{x } 7 \\ \hline \underline{4} \end{array}$$

(carry the 1)

Next you say 7 x 8 = 56
plus 1 carried = __
put down the 7 and carry the ___

$$\begin{array}{r} 182 \\ \text{x } 7 \\ \hline 74 \end{array}$$

(carry the 5)

Finally you say 7 x __ = 7
plus __ carried = 12
put down the 12

$$\begin{array}{r} 182 \\ \text{x } 7 \\ \hline \underline{1}274 \end{array}$$

Now work the following problems: as you work each problem, fill in the blanks.

(A1.) Multiply 683 times 4.

4 x 3 = __

put down the __ and carry the __

4 x 8 = __ plus __ carried = __

put down the __ and carry the __

4 x 6 = __ plus __ carried = __

put down the __

6 8 3
x 4
_ _ _ _ Answer

(A2.) Multiply 159 times 2. (Work this one one your own.)

1 5 9
x 2
_ _ _ Answer

Multiplying by Larger Numbers

Multiplication is harder when the bottom number has more than one digit.

Suppose we were asked to multiply 12 x 43.

We can set the problem up in two ways:

$$\begin{array}{r} 12 \\ \times\ 43 \\ \hline \end{array} \qquad \text{or} \qquad \begin{array}{r} 43 \\ \times\ 12 \\ \hline \end{array}$$

It makes no difference which number is on top when you multiply. Say we do it this way:

$$\begin{array}{r} 12 \\ \times\ 43 \\ \hline \end{array}$$

We have to multiply 12 twice. First by the 3, then by the 4.

Multiplying by 3 we get:

$$\begin{array}{r} 12 \\ \times\ 43 \\ \hline 36 \end{array}$$

Then multiplying by 4 gives:

$$\begin{array}{r} 12 \\ \times\ 43 \\ \hline 36 \\ 48 \end{array}$$

Note that the 48 does not go directly under the 36 but one space over.

Now that we have multiplied the top number by each digit in the bottom number, we add these answers together to get a final answer.

$$\begin{array}{r} 12 \\ \times\ 43 \\ \hline 36 \\ +\ 48 \\ \hline 516 \end{array}$$

Here is the completed answer.

What is the final answer?

____

Now let's multiply 14 by 12.

Here we have multiplied the 14 by 2. You multiply the 14 by 1 then add your answers to get the final answer.

```
   14
 x 12
 ----
   28
 + __

 ____  Answer
```

If your answer was 168, you are correct.

If your answer was 42, you forgot to move the second part to your answer over one space.

Here is what you should have done.

```
   14
 x 12
 ----
   28
 + 14
 ----
  168
```

When you have more than one digit in the number you are multiplying by, you must shift one space each time you multiply.

Now you try this problem.

(A3.) Multiply 13 by 21.

```
    1 3
 x  2 1
 ------
    _ _
  _ _
 ------
  _ _ _
```

Multiplying by Zeros

You can multiply by a zero just as you would by any other number.

```
   25
 x 40
   00
+ 100
 1000
```

Notice that we first had to multiply by zero and then by 4. When we multiplied by 4, we had to shift our answer one space.

Here is another example of multiplying by zero.

```
  111
x 203
```

This time the zero is in the middle of a number. Again, we must multiply by the zero.

```
  111
x 203
  333
 000
+222
22533
```

Notice that we have a row for multiplying by zero as well as by the 2 and the 3.

Why must we bother with a zero? Since zero times any number equals zero, can we just drop the zeros out when we multiply?

No! If we didn't multiply by the zero, we would have:

```
  111
x 203
  333
+ 222
 2553
```

This: a wrong answer

```
  111
x 203
  333
 000
+222
22533
```

Instead of this: a right answer

We don't get the same answer when we drop the zero. The zeros cause us to shift our answer over one space.

Below are a number of problems that have already been worked. Some have been worked correctly, but on others mistakes have been made. For example, on some problems they have forgotten to shift or to multiply by the zero. Look each problem over carefully. If it is correct, mark it with a C. If a mistake has been made, rework the problem and give the correct answer.

(A4.)

```
  115
x 403
-----
  345
 000
 460
-----
 4945     ____________
```

(A5.)

```
   257
 x 101
 -----
   257
  000
 257
 -----
 25957     ____________
```

(A6.)

```
   313
 x 502
 -----
   626
 1565
 -----
 16276     ____________
```

(A7.)

```
   213
 x 150
 -----
   000
 1065
 213       ____________
 -----
 31950
```

(A8.)

```
   611
 x 503
 -----
  1883
  000
 3055
 -----
 32433     ____________
```

(A9.)

```
   222
 x 160
 -----
   000
 1332
 222
 -----
 35520     ____________
```

Now try these problems.

(A10.)

```
   1 4 1
 x 3 0 2
 -------
     _ _ _
   _ _ _
 _ _ _
 -----------
 _ _ _ _ _ _
```

(A11.)

```
     2 2 3
   x 5 0 1
   -------


   ---------
 _ _ _ _ _ _ _
```

(A12.)

```
   2 0 5
 x 4 0 6
 -------


 ---------
 _ _ _ _ _
```

Multiplying by Any Size Number

You can multiply by as large a number as you wish to. The more digits you have in a number, the more rows you will have to add up. As long as you remember to move your answer over one space each time you multiply, you will have no trouble.

(A13.) Multiply 21 by 3064.

We could work this problem like this:

```
      2 1
x 3 0 6 4
---------
      8 4
  1 2 6
  0 0
6 3
---------
_ _ _ _ _
```

Fill in the answer

Try these problems.

(A14.)

```
    2 0 1
  x 4 2 8
  -------
  _ _ _ _
  _ _ _
+ _ _ _
---------
```

(A15.)

```
    4 2 1
  x 1 3 9
  -------


+
---------
```

Multiplying with Zeros on the End of Your Numbers

It is easy to multiply when your numbers end in zeros. Just add on the zeros to the number you multiply.

For example:	You try these:
5 x 10 = 50	(A16.) 54 x 10 = 54___
9 x 100 = 900	(A17.) 6 x 1000 = 6 _____
37 x 1000 = 37000	

To multiply numbers with zeros on the end, remove the zeros, multiply the numbers, then put the zeros back on the end of the answer.

For example:

60 x 20 is: 6 x 2 with (00)

or: 12 with (00) = 1200

Try these problems:

(A18.) 8 x 300 is: 8 x 3 with (00)

or: 24 with (00) = 24___

(A19.) 3 x 30 is: 3 x 3 with (0)

or: ___ with (0) = 9___

(A20.) 20 x 120 is: 2 x 12 with (___)

or: 24 with (___) = _____

(A21.) 70 x 3000 is: 7 x 3 with (_____)

or: _____ with (_____) = _____

LESSON B-I

SHORT DIVISION

Asking How Many Will Fit?

Whenever we divide, we are asking: "How many sets of this number will fit into that number?" For example:

The problem: Dividing 12 by 6 is like asking "How many sets of 6 fit into 12?"

$$\begin{array}{r} 2 \\ 6\overline{)\,12} \end{array}$$

We can say: 12 divided by 6 = 2

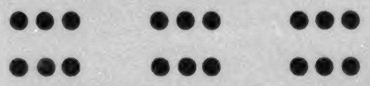
or: there are 2 sets of 6 in 12.

Either way, our answer is 2.

Let's look at another example. Divide 18 by 6.

This is like asking "How many sets of 6 fit into 18?"

$$\begin{array}{r} 3 \\ 6\overline{)\,18} \end{array}$$



We can say: 18 divided by 6 = 3

or: there are 3 sets of 6 in 18

Either way, our answer is 3.

Here is another problem. How many sets of 3 will fit into 15?

$\begin{array}{r} 5 \\ 3\overline{)15} \end{array}$ ●●● ●●● ●●● ●●● ●●●

15 divided by 3 = 5 or there are this many sets of 3 in 15

We can say that there are ___ 3s in 15 because 15 divided by 3 gives 5.

Complete the statements by filling in the blanks.

Example: 21 divided by 7

means: How many 7s will fit into 21?

(B1.) 42 divided by 6

means: How many ___ will fit into 42?

(B2.) ___ divided by 9

means: How many 9s will fit into 81?

A Way to Remember

If you can't remember how many times one number will fit into another, you can write them out. For example:

54 ÷ 9 means: How many 9s will fit into 54?

54 = 9 + 9 + 9 + 9 + 9 + 9 = six 9s

It took six 9s to make 54 -- 9 fits into 54 six times.

Fill in the blanks.

(B3.) $56 \div 8$ means: How many 8s will fit into 56?

$56 = 8 + 8 + 8 + 8 + 8 + 8 + 8$ = seven 8s

So: 8 will fit into 56 ___ times.

(B4.) $42 \div 7$ means: How many 7s will fit into 42?

$42 = 7 + 7 + 7 + 7 + 7 + 7$ = six 7s

So: 7 will fit into 42 ___ times.

Division with Remainders

How many sets of 3 will fit into 6?

6		two 3s
●●●●●●	=	●●● + ●●●

There are two 3s in 6---3 will fit into 6 ___ times.

How many sets of 3 will fit into 7?

7		two 3s	+ 1
●●●●●●●	=	●●● + ●●●	●

When one number will not divide into another number exactly, the amount left over is called a remainder.

We see that 3 will fit into 7 two times, but a remainder of 1 is left over.

Here is an example of a division problem with a remainder.

$$\begin{array}{r} 7 \\ 5 \overline{)\,37} \\ \underline{-35} \\ 2 \end{array}$$

Answer = 7 r2

Our answer tells us that 5 will fit into 37 seven times with 2 left over. Our answer is 7 with a remainder of ___.

Complete each of the following problems.

(B5.)

$$\begin{array}{r} 3 \\ 5 \overline{)\,16} \\ \underline{-15} \\ 1 \end{array}$$

Answer = 3 r___

(B6.)

$$\begin{array}{r} 3 \\ 7 \overline{)\,25} \\ \underline{-21} \end{array}$$

Answer = 3 r___

(B7.)

$$\begin{array}{r} 6 \\ 6 \overline{)\,39} \\ -\underline{\quad} \end{array}$$

Answer = ___ r ___

(B8.)

$$\begin{array}{r} \\ 4 \overline{)\,25} \\ -\underline{\quad} \end{array}$$

Answer = ___ r ___

A Way to Check Your Answer

The remainder (the amount left over) should always be smaller than the number you are dividing by. For example, divide 55 by 6.

Suppose we guess that 6 will go into 55 about 8 times.

```
     8
6/  55
   -48
     7
```

Our remainder is 7. We have too much left over. This means that 6 will fit into 55 more than 8 times. Maybe it will fit 9 times.

```
     9
6/  55
   -54
     1
```

It does fit 9 times. Since our remainder (1) is now smaller than the number we are dividing by (6), we are finished.

Answer = 9 r1

If your remainder isn't smaller than the number you are dividing by, then your answer is too small a number.

Another Check

You can't subtract a larger number from a smaller number. If the number you are subtracting is larger than the number you are subtracting from, your answer is too large.

$$\begin{array}{r} 9 \\ 9\overline{)\,80} \\ \underline{-81} \end{array} \longleftarrow \text{too large} \qquad\qquad \begin{array}{r} 8 \\ 9\overline{)\,80} \\ \underline{-72} \end{array} \longleftarrow \text{okay}$$

Check each of the following problems to see if the answer is too large or too small. If it is worked correctly, mark it with a C. If not, rework the problem and give the correct answer.

(B9.) ____________

$$\begin{array}{rl} 8 & \text{r}10 \\ 9\overline{)\,82} & \\ \underline{-72} & \\ 10 & \end{array}$$

(B10.) ____________

$$\begin{array}{rl} 6 & \text{r}1 \\ 6\overline{)\,37} & \\ \underline{-36} & \\ 1 & \end{array}$$

(B11.) ____________

$$\begin{array}{rl} 4 & \text{r}4 \\ 5\overline{)\,24} & \\ \underline{-20} & \\ 4 & \end{array}$$

(B12.) ____________

$$\begin{array}{rl} 7 & \text{r}9 \\ 8\overline{)\,65} & \\ \underline{-56} & \\ 9 & \end{array}$$

(B13.) ____________

$$\begin{array}{r} 7 \\ 7\overline{)\,48} \\ \underline{-49} \end{array}$$

(B14.) ____________

$$\begin{array}{rl} 5 & \text{r}3 \\ 3\overline{)\,18} & \\ \underline{-15} & \\ 3 & \end{array}$$

Remainders Written as Fractions

Another way to give a remainder is as a fraction. For example:

$$\begin{array}{r} 3 \\ 2\overline{)\,7} \\ \underline{-6} \\ 1 \end{array}$$

We can write our answer like this: 3 r1
or like this: 3 1/2

Notice that in order to write a remainder as a fraction, we placed it over the number we divide by. In the example above, the remainder was 1 and we divided by 2 so we write the remainder as 1/2.

For the following problems, give the answer with the remainder written as a fraction.

Example:

$$\begin{array}{r} 3 \\ 2\overline{)\,7} \\ \underline{-6} \\ 1 \end{array}$$

Answer = 3 1/2

(B15.)
$$\begin{array}{r} 1 \\ 4\overline{)\,5} \\ \underline{-4} \\ 1 \end{array}$$
Answer = 1 1/_

(B16.)
$$\begin{array}{r} 2 \\ 10\overline{)\,25} \\ \underline{-20} \\ 5 \end{array}$$
Answer = 2 _/10

(B17.)
$$\begin{array}{r} 7 \\ 7\overline{)\,50} \\ \underline{-49} \\ 1 \end{array}$$
Answer = 7 _/_

(B18.)
$$\begin{array}{r} 1 \\ 5\overline{)\,8} \\ \underline{-5} \\ 3 \end{array}$$
Answer = __ _/_

(B19.)
$$5\overline{)\,11}$$
Answer = __ _/_

(B20.)
$$8\overline{)\,35}$$
Answer = __ _/_

LESSON B-II

LONG DIVISION

Working in Steps

What happens when we try to divide into a number with several digits?

$$4\overline{)\,52}$$

It's hard to guess how many times 4 will fit into 52.
We have to work this problem in steps like this:

Divide and find the remainder	$\begin{array}{r} 1 \\ 4\overline{)\,5\;2} \\ -4 \\ \hline 1 \end{array}$	Bring down the next digit
Divide and find the remainder	$\begin{array}{r} 1\;3 \\ 4\overline{)\,5\;2} \\ -4 \\ \hline 1\;2 \\ -1\;2 \\ \hline 0\;0 \end{array}$	No more digits to bring down so: problem is finished

Notice that a division problem is worked in two steps.

First you divide and get
a remainder

Second you bring down the
next digit

No matter how large the number you are dividing into, you just keep repeating these two steps.

Now let's try another long division problem. Divide 174 by 3.

We can't divide 3 into 1, so we start with 3 into 17

```
      5
    ______
3/ 1 7 4
  -1 5
   ---
     2
```

Notice the 5 goes over the last digit in 17

1st
Divide 3
into 17

Bring down the next digit which is ____.

When we bring down the 4, we get 24, dividing 3 into 24 gives us 8

```
      5 8
    ______
3/ 1 7 4
  -1 5
   ---
     2 4
    -2 4
     ---
     0 0
```

The remainder is zero so our answer is 58.

2nd
Divde 3
into 24

Here is another long division problem. Divide 1499 by 7.

DIVIDE		BRING DOWN	
First, divide 7 into 14 and get a remainder	2 7/1499 -14 00	Now, bring down the next digit and get 9	2 7/1499 -14 009
Second, divide 7 into 9 and get a remainder	21 7/1499 -14 009 -7 2	Now, bring down the next digit and get 29	21 7/1499 -14 009 -7 29
Third, divide 7 into 29 and get a remainder	214 7/1499 -14 009 -7 29 -28 1	Since there are no more digits to bring down, our answer is: 214 r1 or 214 1/7	

Notice that each time after we divided we brought down the next digit.

Finish each of the following long division problems. Don't forget to bring down the next number each time you divide.

(B21.)
1
3/462
-3
16

(B22.)
7
7/511
-49
2

(B23.)
8/6576

(B24.)
2/3104

Getting Zeros in Your Answer

What do you do in a case like this?

1		
3/315	First, we divided 3 into 3.	Divide
-3	Then we brought down the 1.	
01		Bring down

How many times will 3 go into 01? ______

10		
3/315	3 goes into 01 zero times.	Divide
-3	We must put the zero in	
01	our answer. Then we	
-00	bring down the 5 and get 15.	
15		Bring down

Now we can divide 3 into 15.

105		
3/315	We find that 3 goes into	Divide
-3	15 exactly 5 times. There	
01	are no more numbers to	
-00	bring down. So we are	
15	finished.	
15		
00		Finished

Remember! You bring down only one number each time. If the number you bring down is too small to divide into--you put a zero in your answer.

Each of these problems will have a zero in the answer. Finish each problem.

(B25.)

1
4/424
-4
02

Answer ________

(B26.)

15
2/3012
-2
10
-10
001

Answer ________

(B27.) 5/6025 Answer ________

(B28.) 2/1210 Answer ________

Watching for Errors in Division

In order to do well at long division, you have to be able to recognize when you have chosen a wrong answer. Study the following problems and you will learn how to spot wrong answers.

#1. Answers in the wrong place

Sometimes you may put your answer in the wrong place. Which of the following three examples has the answer in the right place?

Wrong	Right	Wrong
$\begin{array}{r} 7 \\ 4\overline{)308} \\ \underline{-28} \\ 2 \end{array}$	$\begin{array}{r} 7 \\ 4\overline{)308} \\ \underline{-28} \\ 2 \end{array}$	$\begin{array}{r} 7 \\ 4\overline{)308} \\ \underline{-28} \\ 2 \end{array}$

The answer must go above the last digit of the number you are dividing into. Since we start by dividing 4 into 30, our answer of 7 should go above the zero.

Each of the following problems have been worked in three ways. For each problem, circle the example where the answer is in the right place.

Example:	[✓ $\begin{array}{r} 2 \\ 3\overline{)783} \end{array}$]	$\begin{array}{r} 2 \\ 3\overline{)783} \end{array}$	$\begin{array}{r} 2 \\ 3\overline{)783} \end{array}$

(B29.) $\begin{array}{r} 5 \\ 5\overline{)250} \end{array}$ $\qquad$ $\begin{array}{r} 5 \\ 5\overline{)250} \end{array}$ $\qquad$ $\begin{array}{r} 5 \\ 5\overline{)250} \end{array}$

(B30.) $\begin{array}{r} 1 \\ 6\overline{)840} \end{array}$ $\qquad$ $\begin{array}{r} 1 \\ 6\overline{)840} \end{array}$ $\qquad$ $\begin{array}{r} 1 \\ 6\overline{)840} \end{array}$

#2. Forgetting to divide

Sometimes you may forget to divide before you bring down the next number. For example:

15
3/312
-3
12

Wrong!

Here they should have divided 3 into 1 and gotten 0 before they brought down the 2.

Finish the problem as it should be done.

(B31.)
1
3/312
-3
1

#3. Too small an answer

Sometimes the number you guess for an answer will be too small. For example:

8
6/55
-48
7

Wrong

We know 6 will go into 55 more than 8 times because the remainder (7) is larger than the number we are dividing by (6).

If your remainder is larger than the number you divide by, your answer is too small.

#4. Too large an answer

Sometimes the number you guess for an answer will be too large. For example:

```
   7
5/34
 -35 ←wrong!
```

We know 5 won't go into 34 7 times because 35 is too big to subtract from 34.

If you find you have to subtract a larger number from a smaller one, your answer is too large.

Check the following problems. Circle any that have been done incorrectly. Write the correct answer in the space provided.

(B32.)

```
   602
 2/124
  -12
    04
   -04
     0
```

____

(B33.)

```
    25
 5/125
  -10
    25
   -25
    00
```

____

(B34.)

```
    25
5/1025
 -10
    25
   -25
    00
```

____

(B35.)

```
   712
 8/656
  -56
    96
    -8
    16
```

____

LESSON B-III

DIVISION BY LARGE NUMBERS

Division by Guessing

Sometimes we have to divide by large numbers. When numbers are large, we have to guess at how many times one will go into another. You can make a good guess by looking at the first part of the number. For example:

> How many times will 32 fit into 96? Well, 3 goes into 9 three times, so let's guess 3.

```
   3
32/96        3 x 32 = 96
  -96
   00       (3 was a good guess)
```

Here is another example.

> How many times will 25 fit into 75? Well, 2 goes into 7 three times, so let's try _____.

```
   3
25/75        3 x 25 = 75
  -75
   00       (3 was a good guess)
```

Fill in the blanks in the following statements to show how you can use parts of numbers to guess at the answers to division problems.

(B36.) If you are trying to guess how many times 12 will fit into 48, ask: "How many times will 1 fit into _____?"

(B37.) If you are trying to guess how many times 21 will fit into 64, ask: "How many times will ____ fit into 6?"

Below we are trying to guess how many times one large number will go into another. We do this by looking at the first part of the numbers. Fill in the blanks.

	To decide how many times	We can take the first part of the numbers and say:
	231 fits into 1155	2 goes into 11 five times
(B38.)	105 fits into 420	1 goes into ____ four times
(B39.)	315 fits into 945	3 goes into ____ three times
(B40.)	703 fits into 6119	___ goes into 61 eight times

Selecting a Part to Start With

When you have a very large number to divide into, your first step is to decide what part of the number you can start with. For example:

$21/\overline{\underline{30}248976}$	The smallest part 21 will divide into is 30.

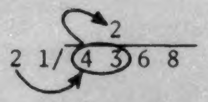
Here are some more examples:

If the problem is:	If the problem is:
$42/\overline{\underline{168}3}$	$83/\overline{\underline{90}57}$
The smallest part we can fit 42 into is 168.	The smallest part we can fit 83 into is 90.

Working a Problem by Guessing

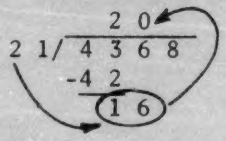
Here is a complete problem worked by guessing. It has been broken down into three steps.

We guess 21 fits into 43 two times because 2 fits into 4 two times.



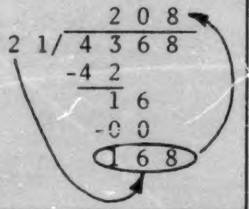
We bring down the 6 and get 16.

Now we guess 21 will fit into 16 zero times because 21 is larger than 16.



We bring down the 8 and get 168.

Now we guess that 21 will fit into 168 eight times because 2 will fit into 16 eight times.



21 fits into 168 exactly eight times so the remainder is zero. There are no more numbers to bring down so we are finished.

Guessing a Single Digit

Can we ever guess 10 as an answer? No! For example:

```
    ___
 21/209
```

We could say: "21 into 209 is like 2 into 20."

Here we have a problem because 2 goes into 20 ten times. But we need a single digit. The closest digit to 10 is 9. So we will try 9 as an answer.

```
       9
    ____
 21/2095
   -189
     20
```

We see that 21 will go into 209 9 times with 20 left over.

Our next step is to bring down the next number which is _____ and get 205. Then we divide by 21.

```
       9
    ____
 21/2095
   -189
     205
```

To decide how many times 21 fits into 205, we ask: how many times will 2 fit into 20?

2 fits into 20 ten times. But, our guess must always be a single digit! The closest digit to 10 is _____. We will guess, then, that 21 fits into 205 nine times.

```
      99
    ____
 21/2095
   -189
     205
    -189
```

9 times 21 gives us 189.

Now we subtract to get our remainder.

When we subtract 189 from 205, we will get a remainder of ____. Now we are finished. We may state that 21 will go into 2095 99 times with a remainder of 16.

Correcting Wrong Guesses

Looking at the first part of numbers will not always give you the right answer, but it will give you an answer that is close. When you make a guess, you have to be ready to see if your answer is too large or too small so you can change it.

Correcting a guess that was too large.

Suppose you wanted to divide 380 by 19.

$19\overline{)380}$ The smallest part of 380 that we could divide 19 into is 38.

19 into 38 is like 1 into 3. Let's try 3.

$$\begin{array}{r} 3 \\ 19\overline{)380} \\ \underline{-57} \end{array}$$

57 is too much

Seeing that 57 is too large tells us that we need to try a smaller number than 3. Try 2.

$$\begin{array}{r} 2 \\ 19\overline{)380} \\ \underline{-38} \\ 00 \end{array}$$

Since 3 was too large, we try 2.

Now we are okay--our answer of 2 is not too large since 19 fits into 38 two times.

Spotting Wrong Guesses

Continue to work each of the following problems. If you find that the problem has been started incorrectly, tell whether the answer given was too big or too small. If the problem was started correctly mark right in the blank space.

Example

2
27/410
-54 2 is too big

(B41.) 3
31/935 ________

(B42.) 8
90/820 ________

(B43.) 3
59/179 ________

(B44.) 6
44/259 ________

Each of the following problems is being worked incorrectly. For some, the answers they are giving are too small or too big. For others, the answers are being placed in the wrong position. After each problem write down the reason for the type of mistake that has been made.

Examples

$$\begin{array}{r} 2 \\ 45\overline{)9005} \\ \underline{-90} \end{array} \quad \underline{\text{wrong place}} \qquad \begin{array}{r} 3 \\ 32\overline{)916} \\ \underline{-96} \end{array} \quad \underline{\text{too big}} \qquad \begin{array}{r} 3 \\ 21\overline{)990} \\ \underline{-63} \end{array} \quad \underline{\text{too small}}$$

(B45.)

$$\begin{array}{r} 2 \\ 65\overline{)1365} \\ \underline{-130} \end{array}$$

____

(B46.)

$$\begin{array}{r} 42 \\ 15\overline{)6030} \\ \underline{-60} \\ 30 \end{array}$$

____

(B47.)

$$\begin{array}{r} 5 \\ 50\overline{)250000} \\ \underline{-2500} \end{array}$$

____

(B48.)

$$\begin{array}{r} 4 \\ 45\overline{)1359} \\ \underline{-180} \end{array}$$

____

(B49.)

$$\begin{array}{r} 5 \\ 28\overline{)1689} \\ \underline{-140} \\ 28 \end{array}$$

____

(B50.)

$$\begin{array}{r} 30 \\ 35\overline{)1085} \\ \underline{-105} \\ 3 \end{array}$$

____

Work the following problems and fill in the blanks.

(B51., B52.)

Divide 490 by 51.

$51\overline{)490}$

(B51.) First you divide 51 into 490. This is like dividing 5 into ________.

(B52.) Finish the problem. Your answer is _______ with a remainder of _______.

(B53. through B56.)

Divide 5150 by 25.

```
    2
25/5150
  -50
   15
```

(B53.) First you divide 25 into ____.

(B54.) 25 fits into 51 about _____ times.

(B55.) 2 times 25 = 50. You subtract 50 from 51 then bring down the 5 to get 15. 25 goes into 15 _______ times.

(B56.) Finish the problem. Your final answer is ______.

(B57.)

Divide 498 by 12.

$12\overline{)498}$

(B57.) Your answer is _______. With a remainder of _____.

Dividing by Even Larger Numbers

It makes no difference how large are the numbers you work with, you still do it the same way. For example:

Divide 3036 by 132. (You fill in the blanks.)

$132\overline{)3036}$ The first thing we do is decide what part of 3036 we can divide 132 into.

(Fill in the following blanks.)

Will 132 fit into 3? ______ Into 30? ______ Into 303? ______

We start, then, by guessing 132 goes into 303 three times. As you can see, three is too large. Let's try two.

$$\begin{array}{r} 3 \\ 132\overline{)3036} \\ \underline{396} \end{array} \quad \text{wrong}$$

Did 132 go into 303 two times? ____

How much is left over when we subtract? ____

$$\begin{array}{r} 2 \\ 132\overline{)3036} \\ \underline{-264} \\ 39 \end{array} \quad \text{right}$$

Our next step is to bring down the _____.

We bring down the 6 and get 396.

We would guess that 132 will go into 396.

_______ times

$$\begin{array}{r} 23 \\ 132\overline{)3036} \\ \underline{-264} \\ 396 \end{array}$$

3 times 132 = 396 So three was a good guess and our answer is 23.

LESSON C-I

BASIC FRACTIONS

What Is a Fraction?

You know what fractions look like.

$\frac{1}{4}$	2/5	7/8	$\frac{3}{4}$	1/2

What do they mean?

A fractions tells you two things.

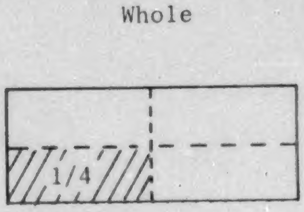
1. It tells you how many parts you are going to divide something into.
2. It tells you how many of those parts you are going to take.

1/4 means

I'm dividing something into 4 parts

and

I'm taking 1 of those parts



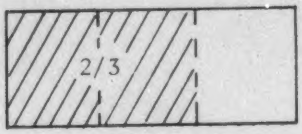
Take 1 of 4 parts

2/3 means

I'm dividing something into 3 parts

and

I'm taking 2 of those parts



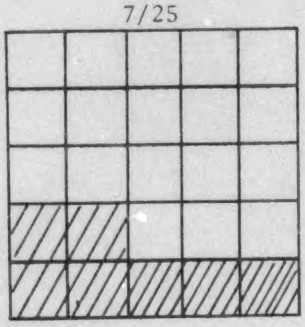
Take 2 of 3 parts

7/25 means

I'm dividing something into 25 parts

and

I'm taking___of those parts



Take 7 of 25 parts

How To Take a Fraction of a Whole

Suppose I have a piece of rope and I want to take 1/4 of it.

I would first divide the rope into 4 equal pieces.

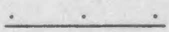
I would then take 1 of the 4 pieces.

I take 1/4 I leave 3/4

Here is another example.

If I want to take 2/5 of a piece of rope, I will first have to divide the rope into _____ parts.

Now that I have the rope divided into 5 parts, how many of these parts do I cut off to get 2/5 of the rope? _______

I take this 

2/5

I leave this

3/5

I had to take 2 of the 5 parts to get 2/5 of the rope.

Fill in the blanks.

(C1.) If I divide a rope into 3 pieces and take 1 piece: I have __/__ of the rope.

(C2.) If I divide a rope into 7 pieces and take 3 pieces: I have __/__ of the rope.

2/3

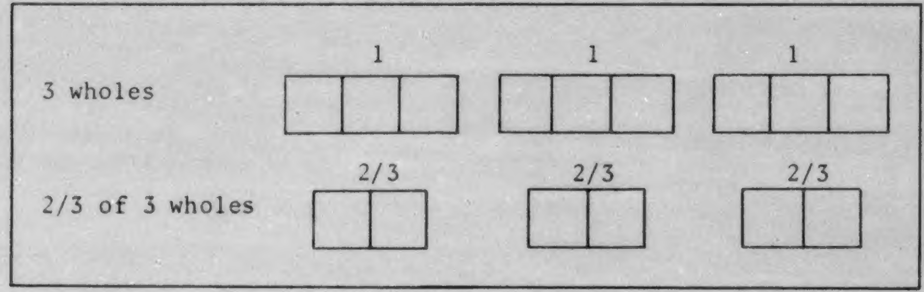
1/3

3/7

4/7

Taking a Fraction of a Number

Suppose I say: Take 2/3 of 3. Do you multiply or divide by 3?



We can see that taking 2/3 of 3 wholes gives us 3 sets of 2/3.

So: 2/3 of 3 is the same as 3 times 2/3

"OF" MEANS "TIMES!"

2/3 of 3	means	2/3 + 2/3 + 2/3	or 3 times 2/3
5/6 of 4	means	5/6 + 5/6 + 5/6 + 5/6	or 4 times 5/6
1/2 of 4	means	1/2 + 1/2 + 1/2 + 1/2	or 4 times 1/2

Look at the above examples carefully. Notice that for each example of has been changed to times.

"Of" means "times." Fill in the blanks to replace of with times.

2/7 of 9   1/4 of 2

means: 2/7 _____ 9   means: 1/4 _____ 2

Multiplying Whole Numbers by Fractions

If 2/3 of 6 means 2/3 times 6, how do we multiply?

$$\boxed{\frac{2}{3} \times 6 = \frac{2 \times 6}{3} = \frac{12}{3}}$$

Here are some more examples; look them over.

$$\frac{1}{2} \times 3 = \frac{1 \times 3}{2} = \frac{3}{2}$$

$$\frac{5}{8} \times 9 = \frac{5 \times 9}{8} = \frac{45}{8}$$

Notice that you multiply the whole number times the top of the fraction.

Be careful not to multiply your whole number by the bottom of your fraction!

$3/4 \times 2 = \frac{3}{4 \times 2}$	wrong!
$3/4 \times 2 = \frac{3 \times 2}{4}$	right!

Finish each of the following problems. (Do not reduce your answer.)

(C3.) $\frac{7}{3} \times 5 = \frac{7 \times 5}{3} = \frac{\_\_}{3}$

(C4.) $3/5 \times 6 = \frac{3 \times 6}{\_} = \frac{18}{\_}$

(C5.) $2/9 \times 7 = \frac{\_ \times \_}{9} = \_\_$

(C6.) $\frac{5}{7} \times 2 = \frac{\_ \times \_}{\_} = \_\_$

(C7.) $4 \times \frac{1}{2} = \frac{\_ \times \_}{\_} = \_\_$

(C8.) $6 \times 1/3 = \frac{\_ \times \_}{\_} = \_\_$

Taking a Fraction of a Fraction

Now you know how to take a fraction of a whole number. You just multiply the top of the fraction by the number. How about if you want to take a fraction of a fraction? For example:

What is 2/3 of 7/8?

We have said that "of" means "times." This still holds.

2/3 of 7/8 means 2/3 times 7/8

so!

$$\frac{2}{3} \times \frac{7}{8} = \frac{14}{24}$$

When you take a fraction of a fraction, you multiply the tops together and the bottoms together.

Finish each of the following problems by multiplying the tops and bottoms together. (Do not reduce your answer.)

(C9.) 6/7 of 2/5 = $\frac{6}{7} \times \frac{2}{5} = \frac{\underline{\hspace{2em}}}{35}$

(C10.) 1/2 of 5/9 = $\frac{1}{2} \times \frac{5}{9} =$ __

(C11.) 1/4 of 6/7 = ____

(C12.) 3/8 of 2/5 = ____

(C13.) 3/4 of 3/4 = ____

(C14.) 8/5 of 2/3 = ____

(C15.) 3/4 x 2/3 = ____

(C16.) 1/4 x 9/2 = ____

Dividing by Fractions

To divide 4 by 2, you ask how many times will 2 fit into 4.

To divide 2 by 1/3, you ask how many times will 1/3 fit into 2.

It is easier to see how many times 2 will fit into 4.
It is harder to see how many times 1/3 will fit into 2.
To make it easier to divide by a fraction, we have a rule.

To divide by a fraction, invert the number you divide by and multiply.

For example:

4 divided by 1/2 = ?

rule: "invert (turnover) the 1/2 and multiply."

4 times $\frac{2}{1} = \frac{4 \times 2}{1} = 8$

Another example:

25 divided by 5/6 = ?

25 times $\frac{6}{5} = \frac{25 \times 6}{5} = \frac{150}{5}$

Finish the following problems. (Do not reduce your answer.)

(C17.) 32 divided by 8/9 = 32 times $\frac{\_}{8}$ = ___

(C18.) 12 divided by 3/4 = 12 times _ = ___

Which fractions must be inverted to work the problems? Circle those fractions that would have to be inverted and underline those fractions that should not be inverted.

Examples:

9 divided by 3/4	10 times 1/2

(C19.) 3 divided by 1/3

(C20.) 8 times 2/3

(C21.) 5 times 4/5

(C22.) 4 divided by 2/5

(C23.) 7 times 3/2

(C24.) 9 divided by 4/3

When you divide a fraction by a fraction, which one do you invert?

2/3 ÷ 1/4 ←Invert this one.

Always invert the fraction you divide by. For example.

Change:	2/3 divided by 1/4	to:	$\frac{2}{3} \times \frac{4}{1}$
Change:	$\frac{5}{8} \div \frac{6}{7}$	to:	$\frac{5}{8} \times \frac{7}{6}$

Remember--you only invert one fraction--the one you divide by!

Change each of the following division problems to a multiplication problem by inverting. Fill in the blanks.

(C25.) Change: 7/9 divided by 2/5 to: $\frac{7}{9} \times \frac{\_}{2}$

(C26.) Change: $\frac{6}{7} \div \frac{3}{8}$ to: $\frac{6}{7} \times \_$

(C27.) Change: 2/3 divided by 2/3 to: _ x _

Dividing a Fraction by a Whole Number

2/3 divided by 2 = 2/3 times 1/2

Remember, you only invert the number you divide by. In the above example, the number we divide by is the _____. How do we invert a whole number like 2?

To invert a whole number, put a one over it. For example:

2 inverted = 1/2 3 inverted = 1/3 25 inverted = 1/25

Why is this true?

2 is equal to $\frac{2}{1}$. If we invert $\frac{2}{1}$, we get $\frac{1}{2}$.

You see, any whole number (like 4) can always be changed to a fraction (like 4/1) then, it can be inverted (to 1/4).

Invert each of the following numbers:

Example: 7 when inverted = 1/7

(C28.) 2/1 when inverted = ___

(C29.) 6 when inverted = ___

(C30.) 9 when inverted = ___

(C31.) 100 when inverted = ___

Invert the following numbers: write your answer in the blank below.

Example:	(C32.)	(C33.)	(C34.)	(C35.)	(C36.)
1/8	5/2	7/1	11	2/3	16/25
8/1	___	___	___	___	___

Which number do you invert when you divide? Remember, you only invert the number you divide by. For each of the following problems, circle the number that you will have to invert before you can work the problem.

Example: 5/6 ÷ [✓ 9/2]

(C37.) 1/4 ÷ 1/8

(C38.) 19 ÷ 2/7

(C39.) $\frac{3}{5} \div \frac{8}{9}$

(C40.) 1/3 ÷ 2

(C41.) a ÷ b/c

(C42.) $\frac{1}{a} \div \frac{1}{b}$

Complete the following rules.

(C43.) You only have to invert when you ________________
(multiply/divide)
with fractions.

(C44.) When you divide you must invert ______________________.
(both fractions/the fraction you divide by)

Work each of the following problems.

Examples:

3/4 divided by 5 = 3/4 ÷ 5/1

or: 3/4 x 1/5 = 3/20

7/8 times 2/3 = 7/8 x 2/3 = 14/24

(C45.) multiply 2/3 by 1/4 __/__ x __/__ = __/__

(C46.) divide 1/8 by 2/7 __/__ ÷ __/__

or: __/__ x __/__ = __/__

(C47.) 6/5 ÷ 1/2 = ____ __/__ ÷ __/__

or: __/__ x __/__ = __/__

(C48.) 3/8 ÷ 2 = ____ __/__ ÷ __/__

or: __/__ x __/__ = __/__

(C49.) 5/4 x 5/9 = ____

(C50.) 7/8 divided by 2 = __/__ ÷ __/__

or: __/__ x __/__ = __/__

(C51.) 6 ÷ 2/9 = ____ x __/__ = ____

Subtracting fractions is like subtracting whole numbers.

$$5 - 2 = 3$$

so:

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$

As in adding fractions, you subtract the tops but the bottom stays the same.

To add or subtract fractions, the bottoms must be the same.

Can you subtract 2/3 from 5/2?

$\frac{5}{2} - \frac{2}{3}$ No, the bottoms are not the same.

Can you subtract 3/5 from 4/5?

$\frac{4}{5} - \frac{3}{5}$ Yes, the bottoms are the same.

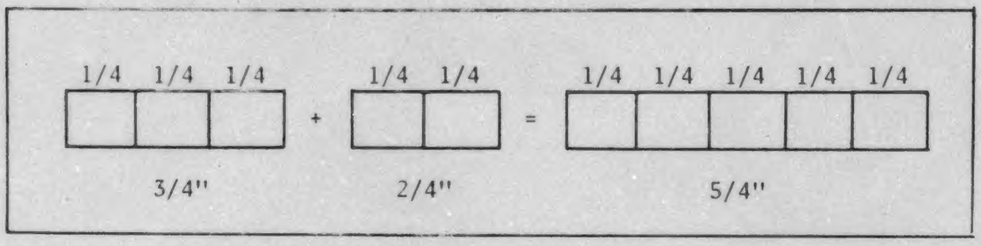
Work these subtraction problems. (Remember, the bottom number stays the same.)

(C54.) $\frac{4}{9} - \frac{2}{9} =$ ____

(C55.) $\frac{11}{8} - \frac{6}{8} =$ ____

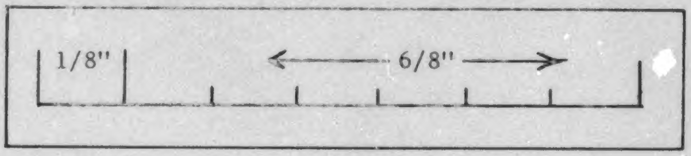
Adding and Subtracting Fractions

Adding fractions is like adding whole numbers.



3 fourths of an inch + 2 fourths of an inch = 5 fourths of an inch

How about if you added 1/8 and 6/8 inches.



1 eighth + 6 more eighths = ______ eighths

1/8 + 6/8 = 7/8	You add the tops but the bottom stays the same.

Work the following problems by adding the tops.

(C52.) $\frac{1}{8} + \frac{2}{8} = \frac{\_\_}{8}$

(C53.) $\frac{1}{9} + \frac{4}{9} =$ ___

LESSON C-11

MIXED NUMBERS

What Is a Mixed Number?

A mixed number has a fraction and a whole number mixed together. Well why did you think they called it a mixed number?

1 3/4	2 5/8	3 1/7	8 2/5

All of these are mixed numbers.

Problems Where You Have To Change Mixed Numbers To Fractions

We do not know how to multiply or divide with mixed numbers.

1 5/8 times 3 1/2 = ?

6 2/3 divided by 7 2/5 = ?

We know how to divide and multiply with fractions, but not with mixed numbers. But, what if we knew how to change a mixed number into a fraction? Our problems would be solved--right?

To change mixed numbers to fractions we change the whole number.

1 3/4 mixed number	=	1 whole number	+	3/4 fraction	=	4/4 fraction	+	3/4 fraction	=	7/4 fraction

In the following pages we will see how to change any mixed number into a fraction.

Changing A Whole Into A Fraction

The trick in changing a mixed number to a fraction is in changing the whole number. We need to know how to change a whole into a fraction.

Changing 1 Into a Fraction

You can change 1 into any fraction with the same top and bottom.

$1 = \frac{2}{2}$	$1 = \frac{3}{3}$	$1 = \frac{4}{4}$
$1 = \frac{5}{5}$	$1 = \frac{6}{6}$	$1 = \frac{7}{7}$

Fill in the top numbers to make these fractions equal to 1.

(C56.) $\frac{\quad}{3}$ (C57.) $\frac{\quad}{9}$ (C58.) $\frac{\quad}{12}$

Now that you know how to change 1 into a fraction, you can change any whole number into a fraction. For example:

$3 = ?/4$

We know that $1 = 4/4$ so: $3 = 4/4 + 4/4 + 4/4$

We can say that 3 is equal to $\frac{3 \times 4}{4}$ or 12/4.

To change a whole into a fraction, multiply the whole times the bottom of the fraction. For example:

$6 = ?/4$ $6 = \frac{6 \times 4}{4} = \frac{24}{4}$	$8 = ?/4$ $8 = \frac{8 \times 4}{4} = \frac{32}{4}$
$6 = ?/5$ $6 = \frac{6 \times 5}{5} = \frac{30}{5}$	$8 = ?/5$ $8 = \frac{8 \times 5}{5} = \frac{40}{5}$

For each of the following problems, a whole number is being changed into a fraction. Complete the changes.

(C59.) 2 = ?/4

$$2 = \frac{2 \times 4}{4} = \frac{8}{\_\_}$$

(C60.) 3 = ?/6

$$3 = \frac{\_\_ \times \_\_}{6} = \frac{\_\_}{\_\_}$$

(C61.) 4 = ?/3

$$4 = \frac{\_\_ \times \_\_}{\_\_} = \frac{\_\_}{\_\_}$$

Changing Mixed Numbers to Fractions

Now that you know how to change a whole number into a fraction, you can change a mixed number to a fraction. A mixed number is like two numbers added together, a whole plus a fraction. For example:

1 3/4 is 1 + 3/4

To change 1 3/4 into a fraction, we change the 1 to 4/4.

1 = 4/4

so

1 3/4 = 4/4 + 3/4

and

4/4 + 3/4 = 7/4

We changed the 1 into ____ and added the 4/4 to 3/4 to get ____.

Here is another example for changing mixed numbers to fractions:

Change 3 1/2 into a fraction

$$3 = \frac{3 \times 2}{2} = \frac{6}{2}$$

so:

3 1/2 = 6/2 + 1/2

and:

6/2 + 1/2 = 7/2

To change the mixed number into a fraction, we had to change the whole number into a fraction.

We found that 3 = _/2.

We then added the 1/2 to get _/2.

For the following problems, mixed numbers are being changed to fractions. Complete the changes. (Do not reduce your answer.)

Example: 7 1/2 = ?/2

well: $7 = \frac{7 \times 2}{2} = 14/2$ so: 7 1/2 = 14/2 + 1/2

and: 14/2 + 1/2 = 15/2

(C62.) 5 3/4 = ?/4

well: $5 = \frac{5 \times 4}{4} = 20/4$ so: 5 3/4 = 20/4 + 3/4

and: 20/4 + 3/4 = __/__

(C63.) 1 3/8 = ?/8

well: 1 = _/8 so: 1 3/8 = _/8 + 3/8

and: 8/8 + 3/8 = __/__

(C64.) 2 5/6 = ?/6

well: $2 = \frac{\underline{\quad}\ \text{x}\ \underline{\quad}}{6} = \frac{\underline{\quad}}{6}$ so: 2 5/6 = _/_ + 5/6

and: _/_ + 5/6 = __/__

(C65.) 3 4/5 = ?/5

well: $3 = \frac{\underline{\quad}\ \text{x}\ \underline{\quad}}{\underline{\quad}}$ = _/_ so: 3 4/5 = _/_ + 4/5

and: _/_ + 4/5 = __/__

Where You Change Mixed Numbers to Fractions

You only have to change mixed numbers to fractions when you multiply or divide fractions. Before you multiply or divide with fractions, first change all mixed numbers to fractions. For example:

$$\begin{array}{r} 1\ 2/3 \\ \underline{\text{x}\quad 4} \end{array}$$

First change 1 2/3
into a fraction.

1 2/3 = 3/3 + 2/3 = 5/3

Now multiply 5/3 by 4.

5/3 x 4 = 20/3 Answer

Here is an example of a division problem.

Divide 9 by 2 1/3

First change 2 1/3
into a fraction.

2 1/3 = 6/3 + 1/3 = 7/3

Now divide 9 by 7/3.

9 ÷ 7/3 can be changed to
9 x 3/7 = 27/7 Answer

```
Here is another example:    2 1/2
                          x 1 1/4
                          -------
```

First change your mixed numbers into fractions.

Change 2 1/2 to 4/2 + 1/2 = __/__

Change 1 1/4 to __/__ + 1/4 = __/__

Now multiply the fractions together.

5/2 x 5/4 = _/_ Answer

To complete the problem, we first had to change 2 1/2 to 5/2, and 1 1/4 to 5/4. Then we multiplied 5/2 times 5/4 to get 25/8.

Where You Do Not Change Mixed Numbers to Fractions

You don't have to change mixed numbers into fractions when you add or when you subtract. For example:

```
  1 1/4      5 7/8
+ 3 2/4    - 2 3/8
-------    -------
  4 3/4      3 4/8
```

Whenever possible, work with the whole numbers separately from the fractions. For example:

Add 3 1/8 to 2 4/8

```
  3       1/8
+ 2     + 4/8
---     -----
  5       5/8      Answer = 5 5/8
```

You complete this example.

Subtract 2 4/7 from 7 6/7.

```
  7       6/7
- 2     - 4/7
---     -----
  _     _____      Answer = 5 2/7
```

Borrowing to Subtract Mixed Numbers

We have said that you don't have to change mixed numbers into fractions when you add or subtract. However, you may have to borrow in order to subtract a mixed number. For example:

```
   5 2/8       We can subtract the whole numbers,
 - 3 5/8       but how can we subtract 5/8 from 2/8?
```

When the fraction you are subtracting is too large, you have to borrow from a whole. Here is how it's done:

If we borrow 1 from 5 1/4, we get 4 1/4.

Now change the 1 into 4/4 and add it back on.

4 1/4 + 4/4 = 4 5/4

so:

5 1/4 can be changed to 4 5/4

Notice that we made the whole number smaller and the fraction part bigger when we borrowed.

Now let's look at a problem where we have to borrow.

```
Change this:                                    To this:

   5 2/8      borrow 1 from 5 = 4                4 10/8
 - 3 5/8      add 1 to 2/8 = 10/8              - 3  5/8
```

Now we can subtract. 4 - 3 = __, and 10/8 - 5/8 = __; so our answer is 1 5/8.

For each of the following problems, you have to borrow in order to subtract. Finish the problems.

Example:	5 1/8 - 1 7/8	borrow 1 from 5 = 4 add 1 to 1/8 = 9/8	4 9/8 - 1 7/8 3 2/8	Answer

(C66.) 4 1/6
- 1 5/6

borrow 1 from 4 = ___
add 1 to 1/6 = ___/6

3 7/6
- 1 5/6
______ Answer

(C67.) 3 2/7
- 2 3/7

borrow 1 from 3 = ___
add 1 to 2/7 = ___/7

2 ___/7
- 2 3/7
______ Answer

(C68.) 9 2/7
- 3/7

______ Answer

(C69.) 6 1/4
- 2 3/4

______ Answer

Below are several different kinds of problems with mixed numbers. Remember as you work them that you have to change mixed numbers into fractions when you multiply or divide and that you may have to borrow to subtract.

(C70.) 1 3/8 x 2 1/2 = ___

(C71.) 2 3/4 ÷ 1 1/4 = ___

(C72.) 2 5/8 + 9 2/8 = ___

(C73.) 1 5/6 ÷ 2 2/3 = ___

(C74.) 9 - 2 1/8 = ___

(C75.) 9 3/7 - 2 5/7 = ___

LESSON C-III

SIMPLIFYING ANSWERS

Generally, on a test, you will be expected to simplify your answer.

1. If it is a top heavy fraction (the top is larger than the bottom) like 8/7 or 9/2.
2. If both top and bottom can be divided by the same whole number: (6/8 can be reduced because both 6 and 8 can be divided by 2 to give 3/4).

Changing Top Heavy (Improper) Fractions into Mixed Numbers

You have probably noticed that sometimes our answer is a top heavy fraction (one where the top is larger than the bottom).

5/4 9/2 7/5 9/4 8/7

These are top heavy fractions.

You can simplify a top heavy fraction by changing it to a mixed number. To change a top heavy fraction to a mixed number, divide the top by the bottom.

To simplify, divide the top by the bottom.

$3/2 = 2\overline{)3} = 1\ 1/2$

$8/3 = 3\overline{)8} = 2\ 2/3$

Simplify the following fractions. Change them to mixed numbers by dividing the tops by the bottoms.

(C76.) 6/5 = 5/‾6‾ = __ _/_

(C77.) 5/2 = _/‾_‾ = __ _/_

(C78.) 8/3 = _/‾_‾ = ____.

Sometimes mixed numbers have top heavy fractions that need to be reduced. For example:

Add 1 3/4 to 2 3/4.

```
  1 3/4
+ 2 3/4
-------
  3 6/4
```

But 6/4 = 1 2/4 so we can simplify our answer.

3 6/4 = 3 + 1 2/4

= 4 2/4

Work the following problems and simplify your answers by changing top heavy fractions to mixed numbers.

(C79.)

```
  3 5/7
+ 2 6/7
-------
  5 11/7 = 6 __/7
```

(C80.)

```
  1 4/5
+ 2 3/5
-------
  3 7/5 = ______
```

Reducing Fractions

Another way to simplify a fraction is to reduce it. Fractions can be reduced if you can divide the top and bottom by the same number.

(C81.) 3/6 can be reduced to 1/2 because both 3 and 6 can be divided by ______.

(C82.) 4/12 can be reduced to 1/3 because both 4 and 12 can be divided by ______.

Can the fraction 2/7 be reduced? _____

> No! There is no number that will go evenly into both 2 and 7. We say that 2/7 has been reduced as far as it will go.

You can reduce a fraction in steps or all at once.

$\frac{24}{32} = \frac{12}{16} = \frac{6}{8} = \frac{3}{4}$ Here we divided several times by 2.

$\frac{24}{32} = \frac{3}{4}$ Here we divided once by 8.

To reduce a fraction, you have to ask yourself, "what number will divide evenly into both top and bottom?"

For the following fractions, first tell what number to divide top and bottom by; then give the answer. (If the fraction can't be reduced, write "can't reduce.") Be sure to divide by as large a number as you can.

Examples:

6/9 ____divide by 3____ Answer: ____2/3____

7/8 ____can't reduce____ Answer: ____7/8____

(C83.) 14/21 ______________ Answer: ________

(C84.) 21/31 ______________ Answer: ________

(C85.) 3/18 ______________ Answer: ________

(C86.) 7/18 ______________ Answer: ________

(C87.) 6/8 ______________ Answer: ________

(C88.) 12/18 ______________ Answer: ________

(C89.) 24/25 ______________ Answer: ________

(C90.) 15/25 ______________ Answer: ________

LESSON C-IV

EQUIVALENT FRACTION PROBLEMS

Rewriting Problems

Sometimes we have to rewrite a problem before we can work it.

2 feet + 1 yard = ?

We can't work this problem because we can't add yards and feet.

2 feet + 3 feet = 5 feet

By changing 1 yard to 3 feet, we can work the problem.

Just as we cannot add yards and feet, we cannot add fractions that have different bottoms. For example:

1/2 + 3/4 = ?

We can't work this problem because we can't add halves and fourths.

2/4 + 3/4 = 5/4

By changing 1/2 to the equivalent fraction 2/4, we were able to work the problem.

Equivalent means equal!

1/2 is equivalent to 2/4 because 1/2 = 2/4.

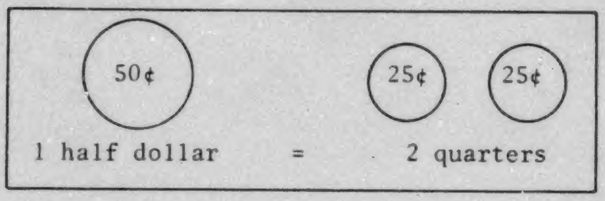
(Just like 1 half-dollar is equal to 2 quarters.)

Changing to Equivalent Fractions

We have said that when we are adding or subtracting fractions and the bottoms are not equal, we have to change to equivalent fractions.

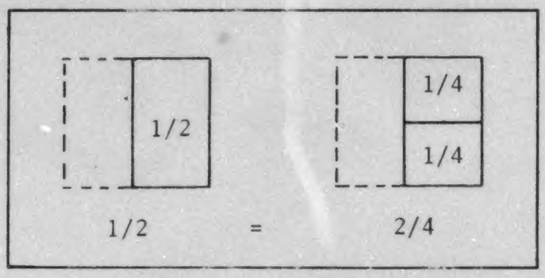
How do we change fractions to equivalent fractions?

Finding an equivalent fraction is writing the same amount in a different way. We can, for example, write the same amount of money in different ways.



Both amounts are equal to 50¢.

We can also write the same fractional amount in different ways.



One half of a square is equal to ____ fourths of a square.

Notice that since 4ths are twice as small as halves, we need twice as many.

Whenever you divide something into parts or fractions, the smaller the parts, the more you have.

3/4 = 6/8 = 9/12

8ths are twice as small as 4ths so we need twice as many.

3/4 = 2 times 3/8 or: _/8

12ths are ____ times as small as 4ths so we need 3 times as many.

3/4 = 3 times 3/12 or: __/12

When do we change fractions into equivalent fractions?

We have to change fractions into equivalent fractions when we are adding or subtracting fractions with different bottoms.

2/3 x 5/8	Do we have to change these fractions?

No! Just multiply the tops and bottoms.

4/9 + 5/9	Do we have to change these fractions?

No! The bottoms are already the same.

5/8 + 3/4	Do we have to change these fractions?

Yes! We need to get the bottoms the same.

For each of the following problems, tell if we have to change the fractions to equivalent fractions to work the problems.

Examples: 3/4 ÷ 1/2 No   3/4 - 1/8 Yes   2/4 + 3/4 No

(C91.) 1/4 + 1/2 ___

(C92.) 3/7 + 5/7 ___

(C93.) 3/8 ÷ 3/4 ___

(C94.) 3/8 x 3/4 ___

(C95.) 9/8 - 1/2 ___

(C96.) 2 3/16 + 5 9/16 ___

How To Make the Change

If we are going to change a fraction into an equivalent fraction with more smaller parts, we need to decide how many times as many parts we need.

For example: 3/4 = ?/8

8ths are 2 times as small as 4ths so we need 2 times as many.

We have 3 fourths, so we need twice as many eights.

So: 3/4 = 2 x 3/8 = 6/8

Here is another example of changing to an equivalent fraction.

Change 2/3 to 12ths. 2/3 = $\frac{4 \times 2}{12}$ = 8/12

In the above example, we are changing 3rds to 12ths. But 12ths are 4 times as small as 3rds so we need _____ times as many 12ths as 3rds. Since we had 2 3rds, we needed 4 x 2 12ths = ____ 12ths.

Complete the following problems, changing each fraction to an equivalent fraction.

(C97.) 1/4 = ?/8

We need 2 times as many 8ths.

$$\frac{1}{4} = \frac{2 \times 1}{8} = \frac{\_\_\_}{8}$$

(C98.) 2/3 = ?/6

We need _____ times as many 6ths.

$$\frac{2}{3} = \frac{2 \times 2}{6} = \frac{\_\_\_}{6}$$

(C99.) Change 3/4 into 12ths.

$$\frac{3}{4} = \frac{\_\_\_ \times 3}{12} = \frac{\_\_\_}{12}$$

(We need ____ times as many 12ths.)

(C100.) 7/8 = how many 16ths?

$$\frac{7}{8} = \frac{\_\_\_ \times 7}{16} = \frac{\_\_\_}{16}$$

(We need ____ times as many 16ths.)

A Fast Method for Finding Equivalent Fractions

You remember that when we change a fraction to an equivalent fraction with smaller parts, we get more parts.

There is a fast way to find an equivalent fraction. To change one fraction into another, all you need is a multiplier. For example:

$\frac{3}{4} = \frac{?}{20}$

Since 4 goes into 20 five times, our multiplier is five.

five times 3 = 15

five times 4 = 20

Answer $\frac{3}{4} = \frac{15}{20}$

Change each of the following fractions into an equivalent fraction by using a multiplier.

Example: 2/7 = ?/21

Since 7 goes into 21 three times, our multiplier is three.

three times 2 = 6

three times 7 = 21

Answer $\frac{2}{7} = \frac{6}{21}$

(C101.) 2/5 = ?/20

Since 5 goes into 20 ____ times, our multiplier is four.

four times 2 = 8

four times 5 = 20

Answer $\frac{2}{5} = \frac{\quad}{20}$

(C102.) 2/3 = ?/15

Since 3 goes into 15 ____ times, our multiplier is ____.

____ times 2 = ___

____ times 3 = 15

Answer $\frac{2}{3} =$ ___

(C103.) 7/8 = ?/16 (Multiplier = 2)

_____ times 7 = ____

_____ times 8 = ____

Answer = _____

(C104.) 6/7 = ?/14 (Multiplier = __)

_____ times 6 = ____

_____ times 7 = ____

Answer = _____

(C105.) 3/4 = ?/8 (Multiplier = __)

_____ x 3 = _____

Answer = __/8

(C106.) 1/3 = ?/9 (Multiplier = __)

_____ x 1 = _____

Answer = __/9

(C107.) 5/6 = ?/12

_____ x 5 = _____

Answer = __/12

(C108.) 2/3 = ?/18

_____ x 2 = _____

Answer = __/__

Problems Where Fractions Are Changed

Below is an example of how you can use equivalent fractions to help you solve a problem.

Add: 2/3 + 5/6 = ?

Well: $\frac{2}{3} = \frac{4}{6}$

So: 4/6 + 5/6 = 9/6

We couldn't add 2/3 to 5/6,
so we changed 2/3 to _/6.

Whenever you add or subtract fractions, the bottoms must be the same. If they are different, use equivalent fractions to get them equal.

For each of the following problems, tell what changes you will have to make before you can work the problem. (Do not work the problems.)

Example: 7/8 - 1/2 = ?

Must change $\frac{1}{2}$ to $\frac{4}{8}$

(C109.) 3/4 + 7/8 = ?

Must change $\frac{3}{4}$ to $\frac{\_}{8}$

(C110.) 5/6 - 2/3 = ?

Must change $\frac{2}{3}$ to $\frac{\_}{6}$

(C111.) 5/8 - 2/16 = ?

Must change $\frac{5}{8}$ to $\frac{\_}{16}$

(C112.) 5/12 + 3/4 = ?

Must change __ to $\frac{\_}{12}$

(C113.) 5/6 + 2/3 = ?

Must change __ to $\frac{\_}{6}$

(C114.) 1/2 - 5/12 = ?

Must change __ to $\frac{\_}{12}$

Finding a Common Bottom for Fractions: Two Different ways.

Is it necessary to change the fractions 3/4 and 2/3 before we can add them? _____

Right! Before we can add or subtract fractions, we must have them both with the same size parts. Both must be in 4ths or 6ths or 12ths, or some other size part, but they must be the same size part. This means that the bottoms of the fractions must be the same.

To get the bottoms of your fractions the same, you need a common number--one that both bottoms will divide into evenly.

For example: 1/2 + 5/6 can be changed to 3/6 + 5/6

2/3 + 3/4 can be changed to 8/12 + 9/12

For 1/2 and 5/6 the common bottom was _____.

For 2/3 and 3/4 the common bottom was _____.

Notice that there are two ways to find a common bottom.

If one bottom will divide evenly into the other, use the largest one as your common bottom. (Change one fraction)

2/3 + 11/12

change to

8/12 + 11/12

Since 3 will divide into 12 evenly,

we use ____ as a common bottom.

We changed 2/3 to ____/12

If one bottom will not divide evenly into the other, multiply the bottoms together to get a common bottom. (Change both fractions)

2/3 - 1/5

change to

10/15 - 3/15

Since 3 will not divide into 5 evenly,

we multiplied 5 x 3 to get ___ as a common bottom.

- Notice that in this case we had to change both fractions.
  We changed 2/3 to __/15 and 1/5 to __/15.

For each of the following problems, you need to find a common bottom for the fractions before you can work the problem. Find the common bottoms. (Do not work the problems.)

Examples:	3/5 - 3/10 = ?	2/3 + 2/5 = ?
	Common bottom = 10	Common bottom = 15
	(5 divides into 10)	(3 times 5 gives 15)

(C115.) 1/2 + 2/6 = ?

Common bottom = ____

(C116.) 6/7 - 5/6 = ?

Common bottom = ____

(C117.) 1/3 + 1/10 = ?

Common bottom = ____

(C118.) 2/3 - 4/15 = ?

Common bottom = ____

Here is a problem where we have to multiply bottoms to get a common bottom.

3/5 - 1/7 = ?

changes to:

$\frac{21}{35} - \frac{5}{35} = ?$

To find a common bottom, we multiply ___ x ___ = 35.

$\frac{3}{5} = \frac{\_\_}{35}$ 35ths are 7 times smaller than 5ths so we need 7 times as many.

$\frac{1}{7} = \frac{\_\_}{35}$ 35ths are 5 times smaller than 7ths so we need ____ times as many.

We had to change 3/5 to 21/35 and 1/7 to 5/35. Our answer is 21/35 - 5/35 = __/35.

Work the following problems.

(C119.) 2/3 + 1/4 = ?

Common bottom = ___

_/12 + _/12 = _/_

(C120.) 3/4 - 2/7 = ?

Common bottom = ___

_/_ - _/_ = _/_

Mixed Numbers

When you add or subtract fractions, you have to have the bottoms the same.

2/3 + 3/4 change to: 8/12 + 9/12

This is true for mixed numbers as well as fractions.

$$\begin{array}{r} 5\ 1/2 \\ \underline{+\ 3\ 1/4} \end{array} \quad \text{change to:} \quad \begin{array}{r} 5\ 2/4 \\ \underline{+\ 3\ 1/4} \end{array}$$

To have the bottoms the same,
we had to change 5 1/2 to 5 _/_.

Work each of the following problems. Use equivalent fractions to get the bottoms of the fractions equal. Simplify all answers.

Example:
6 1/2 (1/2 = 3/6) 6 3/6
- 5 1/3 - 5 2/6
(1/3 = 2/6) 1 1/6

(C121.)
9 7/8 (1/4 = _/8) 9 7/8
- 4 1/4 - 4 _/8
__ _/8 Answer

(C122.)
6 3/4 (3/4 = _/12) 6 _/12
+ 2 2/3 + 2 _/12
(2/3 = _/12) __ _/12 or: __ _/_ Answer

(C123.)
3 1/5 (1/5 = _/_) 3 _/_
+ 2 2/3 + 2 _/_
(2/3 = _/_) __ _/_ Answer

(C124.)
5 2/3 5 _/_
- 1 1/15 - 1 _/_
__ _/_ or: __ _/_ Answer

LESSON C-V

COMPLEX FRACTION PROBLEMS

Subtraction

For some fraction problems, subtraction is more complicated because several steps are required to solve the problem. For example:

$$\begin{array}{r} 5\ 1/4 \\ \underline{-\ 3\ 7/8} \end{array} \quad = \quad \begin{array}{r} 5\ 2/8 \\ \underline{-\ 3\ 7/8} \end{array} \quad = \quad \begin{array}{r} 4\ 10/8 \\ \underline{-\ 3\ \ 7/8} \\ 1\ \ 3/8 \end{array}$$

Our first step was to get the bottoms of the fractions the same, so we changed 5 1/4 to 5 _/_. Next we saw that we couldn't subtract 7/8 from 2/8 so we borrowed and changed 5 2/8 to 4 _/_.

Work the following subtraction problems in steps.

(C125.)
$$\begin{array}{r} 6\ 1/4 \\ \underline{-\ \ \ 2/3} \end{array} \quad = \quad \begin{array}{r} 6\ \_/12 \\ \underline{-\ \ \ \_/12} \end{array} \quad = \quad \begin{array}{r} 5\ \_/\_ \\ \underline{-\ \ \ \_/\_} \\ 5\ \_/\_ \end{array} \text{ Answer}$$

(C126.)
$$\begin{array}{r} 7\ 3/10 \\ \underline{-\ 5\ 1/2} \end{array} \quad = \quad \begin{array}{r} 7\ \_/\_ \\ \underline{-\ 5\ \_/\_} \end{array} \quad = \quad \begin{array}{r} \\ \underline{-\qquad\quad} \end{array} \quad = \quad \underline{\qquad\qquad} \text{ Answer}$$

(C127.)
$$\begin{array}{r} 8\ 2/5 \\ \underline{-\ 2\ 2/3} \end{array} \quad = \quad \begin{array}{r} \\ \underline{-\qquad\quad} \end{array} \quad = \quad \begin{array}{r} \\ \underline{-\qquad\quad} \end{array} \text{ Answer}$$

Addition and Multiplication

For some problems, you may have more than two fractions to combine. For example:

2 1/3 1 3/4 + 2/5	or	1 1/3 x 4/5 x 2 1/2 = ?

To work these types of problems, just take two fractions at a time.

Addition with More than Two Fractions

Here is an example of an addition problem worked in steps.

2 1/3 + 1 3/4 + 2/5 = ?

Step 1		Because:
Change:	(2 1/3 + 1 3/4) + 2/5	2 1/3 + 1 3/4 =
To:	4 1/2 + 2/5	2 4/12 + 1 9/12 =
		3 13/12 or 4 1/12
Step 2		Because:
Change:	4 1/12 + 2/5	4 1/12 + 2/5 =
To:	4 29/60	4 5/60 + 24/60 =
		4 29/60

In Step 1, we added the first two fractions together to get 4 1/12.

In Step 2, we added on 2/5 to get our final answer of 4 29/60.

Multiplication with More than Two Fractions

Suppose we are given the problem: multiply 1 1/3 x 4/5 x 2 1/12.

Step 1

Change: (1 1/3 x 4/5) x 2 1/2

To: ____________ x 2 1/2

Because:

1 1/3 x 4/5 =

4/3 x 4/5 =

16/15 (first answer)

Step 2

Change: 16/15 x 2 1/2

To: 2 2/3

Because:

16/15 x 2 1/2 =

16/15 x 5/2 =

80/30 or 2 2/3 (final answer)

Work the following fraction addition and multiplication problems.

(C128.) 3/10 x 4/5 x 5/8 = ?

3/10 x 4/5 = ____ (1st answer)

____ x 5/8 = ____ or ____(final answer)

(C129.) 5 1/2 + 2 3/4 + 7 2/3 = ?

5 1/2 + 2 3/4 = ____ (1st answer)

____ + 7 2/3 = ____ or ____(final answer)

(C130.) 2 1/2 x 2/3 x 1 3/5 = ?

2 1/2 x 2/3 = ____ (1st answer)

____ x 1 3/5 = ____ or ____ (final answer)

(C131.) 2 1/8 + 3/5 + 3 1/3 = ?

2 1/8 + 3/5 = ____ (1st answer)

____ + 3 1/3 = ____ (final answer)

Work each problem, one step at a time, on separate paper. Place your answer in the blank space.

(C132.)
3 1/4
4 1/3
+ 6 1/5
____ Answer

(C133.)
7 1/12
- 3 1/3
____ Answer

(C134.) Multiply 1 1/4 x 5/8 x 3 1/2 ____ Answer

(C135.)
10 5/8
- 4 5/7
____ Answer

(C136.)
1 3/4
- 4/5
____ Answer

(C137.) Add 2 1/4 + 7 5/6 + 4 3/8 ____ Answer

(C138.) 2 2/5 x 5/8 x 1 2/3 = ? ____ Answer

Simplifying Complex Problems

Sometimes a complex fraction problem can be simplified so it is easier to work. For example:

$$\frac{25}{5} \times \frac{21}{3} = \frac{525}{15} = \frac{105}{3} = 35$$

could have been worked like this:

$$\frac{5}{1} \times \frac{7}{1} = 35$$

By recognizing that 25/5 can be changed to 5/1, and that 21/3 can be changed to __/1 we can simplify the problem.

Simplify each of the following problems by filling in the blanks.

Example: $\frac{28}{4} \times \frac{5}{80} = ?$

$$\frac{7}{1} \times \frac{1}{16} = \frac{7}{16}$$

(C139.) 28/7 x 5/35 = ?

__/1 x __/7 = 4/7

(C140.) 18/81 ÷ 2/8 = ?

18/81 x __/__ = ?

__/9 x __/1 = 8/9

Multiplying and Cross-Simplification

You have just seen how a fractions problem can sometimes be simplified by simplifying the fractions before the problem is worked. For multiplication problems, you can also simplify across fractions. For example:

1/14 x 7/4 = ?

can be simplified as follows:

$$\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8}$$

Note that both the 7 and 14 were divided by 7 to simplify the problem.

You can cross-simplify multiplication problems by dividing the top of one fraction and the bottom of another by the same value.

Simplify each of the following complex problems by filling in the blanks.

Example:

5/36 x 7/5 x 9/11 = ?

$$\frac{5}{36} \times \frac{7}{5} \times \frac{9}{11} = \frac{1 \times 7 \times 1}{4 \times 1 \times 11} = \frac{7}{44}$$

(C141.) 99/100 x 10/99 = ?

$$\frac{\underline{\quad}}{10} \times \frac{1}{\underline{\quad}} = \frac{1}{10}$$

(C142.) 15/14 x 22/9 x 14/11 = ?

$$\frac{\underline{\quad}}{1} \times \frac{\underline{\quad}}{3} \times \frac{\underline{\quad}}{1} = \frac{10}{3}$$

(C143.) 27/77 ÷ 6/7 = ?

27/77 x 7/6 = ?

$$\frac{\underline{\quad}}{11} \times \frac{\underline{\quad}}{2} = \frac{\underline{\quad}}{22}$$

(C144.) 3/100 ÷ 60/25 = ?

3/100 x __/__ = ?

$$\frac{1}{4} \times \underline{\quad} = \underline{\quad}$$

(C145.) 5/8 x 7/3 x 8/7 x 3/5 = ?

__ x __ x __ x __ = ____

(C146.) 100/69 x 3/10 x 7/14 = ?

__ x __ x __ = ____

LESSON D-I

DECIMAL ADDITION, SUBTRACTION, AND MULTIPLICATION

What Is a Decimal?

You use fractions when you want to work with something that has been divided into parts. For example:

1/2 a pie	1 3/4 inches

You can also use decimals when you work with things divided into parts.

Decimal numbers have two parts to them--a whole number part, and a decimal part. Take the number 2.5 for example:

The whole number part = 2

The decimal part = .5

The decimal part of a number is always smaller than a whole.

.5 means that we have 5 out of 10 parts of a whole.

2.5 means we have 2 wholes + 5 out of 10 parts of another whole.

You can have as many places behind the decimal point as you wish. In the example below, you can see how the value of our decimal gets smaller as we move the 5 out more places behind the decimal point.

1 place	.5	=	5/10
2 places	.05	=	5/100
3 places	.005	=	5/1000

Just as 5/100 is smaller than 5/10, .005 is ________ than .5.

Adding and Subtracting Decimals

Whenever you add or subtract decimal numbers, be sure and keep your decimal points lined up.

Here is how we set up decimal addition and subtraction problems.

Add 1.5 to 1.25

```
Set it up                  Not like
like this:                  this:

  1.5                         1.5
+ 1.25                     + 1.25
```

No matter what your numbers are, you should keep the decimal points lined up. Here is another example:

Add .3, 1.4, and .35.

Notice that the decimal points are lined up and carried down into the answer.

```
   .3
  1.4
+  .35
  2.05
```

In subtraction, just like in addition, you must keep the decimal points lined up. See the example below.

Subtract .05 from 1.2.

```
a.   1.2                   b.   1.2
   -  .05                      - .05
```

Which is correct, a. or b.? ______

Right, the decimal points must be lined up.

Finish the following problems by bringing down the decimal point into the answer.

(D1.) 25.20
+ .05
25 25

(D2.) 3.07
- .40
2 67

Set up these problems and find the answer.

(D3.) Add 1.2 and .9

1.2
+ ____
equals ____

(D4.) Subract 1.05 from 1.14

1.14
- ____
equals ____

Missing Zeros

Sometimes in order to set up decimal problems, we have to add on zeros to some of the numbers. For example:

Subtract .03 from 1.1

change:
$$\begin{array}{r} 1.1 \\ -\ .03 \\ \hline \end{array}$$

to:
$$\begin{array}{r} 1.10 \\ -\ .03 \\ \hline 1.07 \end{array}$$

There is no number above the 3, so how can we subtract?

All we had to do was add a zero to change 1.1 to ______.

When we subtracted .03 from 1.10, we got ______ as an answer.

We can always add on the zeros to the end of decimal numbers without changing their values.

What if we have to subtract a decimal from a whole number?

5 - 1.3 = ?

$$\begin{array}{r} 5.0 \\ -\ 1.3 \\ \hline 3.7 \end{array}$$

You can always add a decimal point and zeros to the end of a whole number without changing its value.

5 is equal to 5.0

Set up each of the following problems as if you were going to work it. Add decimal points and zeros where necessary. (Do not work the problems.)

Examples: Add .03 to 5

5.00
+ .03

Subtract 1.5 from 7

7.0
- 1.5

(D5.) Add 1.23 to 5.7

5.7
+ .

(D6.) Subtract .03 from 8.2

8.2
-

(D7.) Add .01, 56, and 2.9

.
.
+ .

(D8.) Subtract 1.79 from 18

-

Work the following problems. Add decimal points and zeros where needed.

(D9.)
102.0
- 1.5
______ Answer

(D10.)
25
2.6
+ .0001
________ Answer

(D11.)
67
- .0051
________ Answer

Multiplying with Decimals

When we add with decimals we have to line up the decimal points. We don't have to line up the decimal points when we multiply.

Must line up for addition.	Don't have to for multiplication.
25.3	29.3
+1.78	x.18

Multiplying with decimals is just like multiplying with whole numbers--except--you have to decide where the decimal goes in your answer. Here is how we decide:

.24
x .5
.120

Since three of the numbers we were multiplying were behind decimals, three of the numbers in our answer had to be behind a decimal point.

Notice that when we multiply we do not line up decimal points.

Here is an example of a problem where we have to decide how many decimal places to have in the answer.

	1 3.2	1 decimal place
	x .4 3	2 decimal places
Answer	5.6 7 6	3 decimal places

We had 1 digit behind a decimal in the top number and

2 digits behind a decimal in the other number so we had

to have _____ digits behind the decimal in our answer

(3 decimal places).

What happens when you have more decimal places than digits?

For example:	.12 x .5 60

We need to have 3 decimal places in our answer but we only have 2 digits.

When you need to have more digits behind the decimal point in your answer, place zeros in front of your answer.

Do this:		Not this:	
	.12 x .5 .060		.12 x .5 .600
← 3 places over		3 places over →	

Finish each of the following problems. Find how many decimal places you need in your answer and add zeros where necessary.

Examples:

1.6	(1 place)	.36	(2 places)
x .3	(1 place)	x .2	(1 place)
.48		.072	
	(2 places)		(3 places)

(D12.)
2.3 (1 place)
x 1.4 (1 place)
9 2
2 3
3 2 2 (__ places)

(D13.)
.2 (1 place)
x .4 (1 place)
— — (__ places)

(D14.)
.2 7 (__ places)
x .3 (__ place)
— — — (__ places)

(D15.)
3 1.6 (__ place)
x .1 1 (__ places)
3 1 6
3 1 6
— — — — (__ places)

(D16.)
2.1
x .4
— —

(D17.)
.4 3
x .3
— — —

(D18.)
6.1
x 1.2

(D19.)
.24
x .33

LESSON D

DECIMAL DIVISION

Dividing Decimals

When we multiplied by decimal numbers we found it was very much like multiplying by whole numbers except that you had to decide where to put the decimal point in your answer.

Dividing by decimals is also very much like dividing by whole numbers. But, again, you have to know where to put the decimal point in your answer.

Here are some examples of dividing with decimal numbers.

$\begin{array}{r} .124 \\ 5/\overline{\;.620} \end{array}$ $\begin{array}{r} 1.4 \\ 3/\overline{\;4.2} \end{array}$ $\begin{array}{r} 20.5 \\ 2/\overline{\;41.0} \end{array}$ $\begin{array}{r} .0002 \\ 6/\overline{\;.0012} \end{array}$

Notice that the decimal point in the answer is always placed above the decimal point in the number being divided.

Put the decimal points into the answers for the following problems. (Remember, the decimal point in your answer must go directly above the decimal point in the number that is being divided.)

(D20.) $\begin{array}{r} 6 \\ 6/\overline{\;3.6} \end{array}$ (D21.) $\begin{array}{r} 005 \\ 5/\overline{\;.025} \end{array}$ (D22.) $\begin{array}{r} 2\ 5 \\ 3/\overline{\;7.5} \end{array}$

Putting zeros in your Decimal Answer

When you divide with decimals, you have to be careful to put zeros in front of your answer when necessary. Look at the difference between these two problems:

$\begin{array}{r} 9 \\ 5/\overline{\;45} \end{array}$ $\begin{array}{r} .09 \\ 5/\overline{\;.45} \end{array}$

When you divide 45 by 5 you don't bother to say 5 goes into 4 zero times. When you divide .45 by 5 you do have to put in the zero or your answer will be wrong.

When you divide into a decimal number, write in all your zeros. For example:

$$\text{Divide .0055 by 5} = 5\overline{)\,.0055}^{\;.0011}$$

Complete the answers to the following problems by filling in zeros.

(D23.) $\overset{.\;\;9}{9\overline{)\,.81}}$

(D24.) $\overset{.\;\;\;\;22}{6\overline{)\,.00132}}$

(D25.) $\overset{.\;4}{32\overline{)\,1.28}}$

Finish the following problems. Add zeros to an answer where necessary.

(D26.) Divide .046 by 2

$$\begin{array}{r} .\;\;23 \\ 2\overline{)\,.046} \\ \underline{-4} \\ 06 \\ \underline{-6} \end{array}$$

Answer = ____________

(D27.) Divide .0075 by 5 $\quad 5\overline{)\,.0075}$

Answer = ____________

(D28.) Divide .36 by 6 $\quad 6\overline{)\,.36}$

Answer = ____________

(D29.) Divide .0020 by 5 $\quad \overline{)\qquad\quad}$

Answer = ____________

(D30.) Divide .224 by 4 $\overline{)\quad\quad}$

Answer = ____________

(D31.) Divide .500 by 25 $\overline{)\quad\quad}$

Answer = ____________

Using Zeros to Get Rid of Remainders

We have said that you can add on zeros to the end of a decimal number without changing the value of the number.

.0 = .00000

5.0 = 5.00000

2.1 = 2.10000

You can see that zeros added on to the end of a decimal number do not change its value.

Sometimes we need to add on zeros to get rid of remainders. For example:

$$\begin{array}{r} .04 \\ 5\overline{).21} \\ -\underline{20} \\ 1 \end{array}$$

What do we do with the 1 that's left over?

$$\begin{array}{r} .042 \\ 5\overline{).210} \\ -\underline{20} \\ 10 \\ -\underline{10} \end{array}$$

By changing .21 to .210, we made the answer come out even.

Finish the problems, adding on enough zeros so your answer comes out to an even number.

```
           4                              5
(D32.)  4/ .17               (D33.)  4/ .21
          -16                          -20
            1                            1
```

Changing Whole Numbers into Decimals

When we divide with whole numbers we sometimes have a remainder. Instead of giving an answer with a remainder, we can give a decimal answer. Here is the same problem worked two ways:

Remainder Answer	Decimal Answer

`  1  r1`
`2/ 3`
` -2`
`  1`
	`   1.5`
	`2/ 3.0`
	`  -2`
	`   1 0`
	`  -1 0`
	`   0 0`

In order to get a decimal answer we added a decimal point to the 3 to get 3.0.

Get a decimal answer for the following problems by adding decimal points and zeros.

(D34.) 4/ 5 (D35.) 5/ 16

(D36.) 12/ 30 (D37.) 4/ 7

By using decimals you can divide a larger number into a smaller one. For example:

Divide 6 by 8 $\quad 8\overline{)\,6\,}$

It's clear that 8 can't go into 6. But what if we add on a decimal point and some zeros?

We can say 6 is equal to 6.00 so:

$$\begin{array}{r} .75 \\ 8\overline{)\,6.00} \\ -5\;6 \\ \hline 40 \\ -\;40 \\ \hline 00 \end{array}$$

Now we can work the problem as if we were dividing 8 into 600.

Get a decimal answer for the following problems by adding on decimal points and zeros.

Divide 3 by 6 $\quad 6\overline{)\,3\,} \;=\; \begin{array}{r} .5 \\ 6\overline{)\,3.0} \\ -3\;0 \\ \hline 0\;0 \end{array}$

(D38.) Divide 3 by 4 $\quad 4\overline{)\,3\,} \;=\; 4\overline{)\qquad\qquad}$

(D39.) Divide 4 by 20 $\quad 20\overline{)\,4\,} \;=\; 20\overline{)\qquad\qquad}$

(D40.) Divide 16 by 32 $\quad \_\_\overline{)\,\_\_\,} \;=\; 32\overline{)\qquad\qquad}$

Changing Between Fractions and Decimals

Fractions to Decimals

Changing fractions to decimals is like division. To change a fraction into a decimal, divide the top number by the bottom number. For example:

3/4 = 3 divided by 4 = 4/ 3

We can't divide 4 into 3, but we can change the 3 into 3.00.

```
    .75
4/ 3.00
  -2 8
    20
   -20
    00
```

We may say, then, that 3/4 means

4/ 3 which gives a decimal answer

of . ________.

Change these fractions into decimals:

Example

7/10 = 10/ 7.0 = .7

(D41.) 1/2 = 2/ 1.0 = ______

(D42.) 4/5 = 5/ ______ = ______

(D43.) 2/5 = _/ ______ = ______

(D44.) 3 1/2 = ?

1/2 = 2/ 1 = 0.______

so: 3 1/2 = 3.______

Decimals to Fractions

We just saw how to change from a fraction to a decimal, here is a rule for changing from a decimal to a fraction.

For each place you have behind your decimal point you should have a zero on the bottom of your fraction. For example:

From Decimal		To Fraction	
.055	← 3 places	$\frac{5}{1,000}$	← 3 zeros
.1350	← 4 places	$\frac{1,350}{10,000}$	← 4 zeros
1.5	← 1 place	$1\frac{5}{10}$	← 1 zero

Change each decimal into a fraction. (Remember to keep whole numbers separate when changing decimals to fractions.)

(D45.) .5 ← 1 place = $\frac{\_\_}{10}$ ← 1 zero

(D46.) .03 ← 2 places = $\frac{3}{\_\_}$ ← 2 zeros

(D47.) .700 = ___

(D48.) 62.1 = 62 ___

Dividing By A Decimal Number

Dividing a decimal number by a whole number is no problem.

```
    1.2
3/ 3.6
  -3
    6
   -6
```

However, when we divide by a decimal number it changes our answer.

```
.3/ 3.6  = ?
```

Before we can divide by a decimal number, we have to change the problem.

Changing numbers without changing answers.

```
We get the same answer if            2
we change this: ----------->     3/ 6

                                     2
to this: ------------------->   30/ 60

                                      2
or this: ------------------->  300/ 600
```

Each time we added on a zero we made our number 10 times larger but since we made both numbers 10 times larger our answer stays the same when we divide.

We can also make a number ten times as large by moving a decimal point.

```
10 times .3 = 3.

10 times 1.4 = 14.

10 times 3.78 = 37.8
```

Notice that to make the number 10 times larger, you move the decimal point to the right.

Dividing By a Decimal

You cannot divide by a decimal. In order to work the following problems, the decimal divisors must be changed to whole numbers before you divide.

Can't Divide	Can Divide
$.5/\overline{1.5}$	$5/\overline{15}$ → →
$.3/\overline{.69}$	$3/\overline{6.9}$ → →
$.25/\overline{.525}$	$25/\overline{52.5}$ ↠ ↠
$4.21/\overline{.1236}$	$421/\overline{12.36}$ ↠ ↠

Notice that we move the decimal over enough so that we are dividing by a whole number.

We move the decimal the same number of places for both numbers.

For the problems below, tell how many places the decimal will have to be moved so that we can divide by a whole number.

Examples:		
	$.32/\overline{.0064}$	two places
	$.005/\overline{1.0525}$	three places

(D49.) $.6/\overline{.24}$ ____ place(s)

(D50.) $3.2/\overline{6.464}$ ____ place(s)

(D51.) $.03/\overline{.975}$ ____ place(s)

(D52.) $13.3/\overline{2.66}$ ____ place(s)

For each of the following problems, move the decimal over enough places so you can divide by a whole number (do not work the problems). Remember that you are moving the decimals to make your numbers larger and that you must move the same number of places for both numbers.

Example: Change .03/ .699 to: 3/ 69.9

(D53.) Change .13/ .3926 to: 13/ __.__

(D54.) Change 3.4/ 1.36 to: 34/

(D55.) Change .4/ 2.88 to: _/

Not Enough Places

We have said that we have to move the decimal over the same number of places for both numbers when we divide. But what if the number we are dividing doesn't have enough places?

.24/ 4.8 or .3/ 6

For both of the above problems we need more places so we can move our decimal over.

When you need more places, you can always add a decimal point and zeros. You should make sure you have enough places before you start the problem. For example:

Change: .24/ 4.8

to: .24/ 4.80

Change: .3/ 6

to: .3/ 6.0

To change .24 to 24 we have to move the decimal ___ place. Since 4.8 has only one decimal place we had to add a zero.

To change .3 to 3 we have to move the decimal ___ places. Since 6 has no decimal places we had to add a decimal point and a zero.

For each of the below problems you need more decimal places before you can change the decimal you divide by into a whole number. First add on enough decimal places, then move the decimals over so you can divide.

Example: .06/$\overline{30}$ changes to: .06/$\overline{30.00}$ or: 6/$\overline{3000}$

(D56.) .25/$\overline{7.5}$ changes to: .25/$\overline{7.5\_}$ or: 25/$\overline{\quad\quad}$

(D57.) .04/$\overline{8}$ changes to: .04/$\overline{8.\_}$ or: 4/$\overline{\quad\quad}$

(D58.) .16/$\overline{3.2}$ changes to: .16/$\overline{\quad\quad}$ or: _/$\overline{\quad\quad}$

(D59.) 1.5/$\overline{30}$ changes to: 1.5/$\overline{\quad\quad}$ or: _/$\overline{\quad\quad}$

(D60.) .003/$\overline{2.17}$ changes to: .003/$\overline{\quad\quad}$ or _/$\overline{\quad\quad}$

Work each of the following problems. Remember, before you can divide you have to move your decimal points so you can divide by a whole number.

Example: .4/$\overline{.8}$ = 4/$\overline{8}$ = 2

(D61.) .5/$\overline{1.0}$ = 5/$\overline{\quad\quad}$ = ________

(D62.) .3/$\overline{.96}$ = 3/$\overline{\quad\quad}$ = ________

(D63.) .02/$\overline{.804}$ = _/$\overline{\quad\quad}$ = ________

(D64.) .025/$\overline{25.000}$ = ___/$\overline{\quad\quad}$ = ________

(Hint: add on a decimal point and zeros where necessary.)

(D65.) $.02\overline{)\,4\;}$ = $.02\overline{)\,4.\underline{\quad}\;}$ = $2\overline{)\qquad}$

(D66.) $.03\overline{)\,.9\;}$ = $.03\overline{)\,.9\;}$ = $\underline{\quad}\overline{)\qquad}$

(D67.) $2.5\overline{)\,5\;}$ = $2.5\overline{)\qquad}$ = $\underline{\;}\overline{)\qquad}$

(D68.) $.15\overline{)\,4.5\;}$ = $.15\overline{)\qquad}$ = $\underline{\quad}\overline{)\qquad}$

LESSON E

UNDERSTANDING MATH SYMBOLS

Can You Speak Mathematics?

Mathematics is like a language. If you can't understand what it means, then you can't do anything with it.

One reason that it is hard to understand a language is that there are so many different ways to say the same thing.

In English, if I wanted to say "I like that," I might say it was good, or nice, or great, or tremendous, or wonderful, or keen, cool, crazy, wild, groovy, or boss.

In math, if I want to divide 2 by 3, I could write it as

$$2 \div 3 \quad \text{or} \quad 3\overline{)\,2} \quad \text{or} \quad 2/3 \quad \text{or} \quad \frac{2}{3} \quad \text{or} \quad 1/3 \times 2$$

All mean the same thing

Study the symbols in the box below. All of them are used in math books and tests so you have to understand what they mean.

Examples

This means add 2 and 3 $\underline{2 + 3}$

This means subtract 2 from 3 $\underline{3 - 2}$

Each of these mean multiply 2 times 3

$\underline{2 \times 3}$ $\quad\underline{(2)(3)}$ $\quad\underline{2 \cdot 3}$ $\quad\underline{2(3)}$

Each of these mean divide 3 by 2

$\underline{2\overline{)\,3}}$ $\quad\underline{3 \div 2}$ $\quad\underline{3/2}$ $\quad\underline{\frac{3}{2}}$

Division - Which Number Do You Divide By?

In order to divide, you have to know what number to divide by. For example:

> 3 ÷ 2 reads as three divided by two. Since we are dividing by 2, we set the problem up like this: $2\overline{)\,3}$

Fill in the blanks. For each of the problems below indicate what number you must divide by, then set the problem up as in the above example.

(E1.) 4 ÷ 2 We must divide by 2 like this: $2\overline{)\,\_}$

(E2.) 9 ÷ 3 We must divide by __ like this: $\_\_\_\_\overline{)\,9}$

(E3.) 15 ÷ 5 We must divide by __ like this: $\_\_\_\_\overline{)\,\_}$

(E4.) 6 ÷ 9 We must divide by __ like this: $\_\_\_\_\overline{)\,\_}$

Now work these problems. If you can't remember what the symbols mean, check back with the examples.

(E5.) $\frac{6}{3} = \_\_\overline{)\,6} = \_\_\_\_$

(E6.) (8)(2) = ____

(E7.) 9 ÷ 3 = ____

(E8.) 4 · 5 = ____

(E9.) 10/5 = ___ ÷ ___ = ___

(E10.) (7)5 = ____

(E11.) 8/2 = ____

(E12.) 6 ÷ 3 = ____

(E13.) (6)(3) = ____

(E14.) $3\overline{)\,9} = \_\_\_\_$

Writing Division Problems as Fractions

Division problems can be written as fractions and fractions can be written as division problems. For example:

$6 \div 2 = 6/2$ $1 \div 3 = 1/3$

$7/\overline{8} = 8/7$ $6/\overline{5} = 5/6$

Check back with these examples as you fill in the blanks below.

Change these division problems into fractions.

(E15.) $5/\overline{7}$ = 7/_ (E16.) $2/\overline{1}$ = _/_

(E17.) $7 \div 8$ = _/_ (E18.) $9 \div 2$ = _/_

(E19.) $4/\overline{13}$ = _/_ (E20.) $13 \div 4$ = _/_

Change these fractions into division problems. **(Do not solve)**

(E21.) 2/3 = $3/\overline{}$ (E22.) 1/4 = 1 ÷ __

(E23.) 5/4 = __ ÷ __ (E24.) 6/2 = $\_\_/\overline{}$

(E25.) 1/7 = $\_\_/\overline{}$ (E26.) 3/5 = __ ÷ __

Some Special Symbols

In addition to the symbols used for addition, subtraction, multiplication, and division, there are special symbols which call for special operations.

Squaring a Number

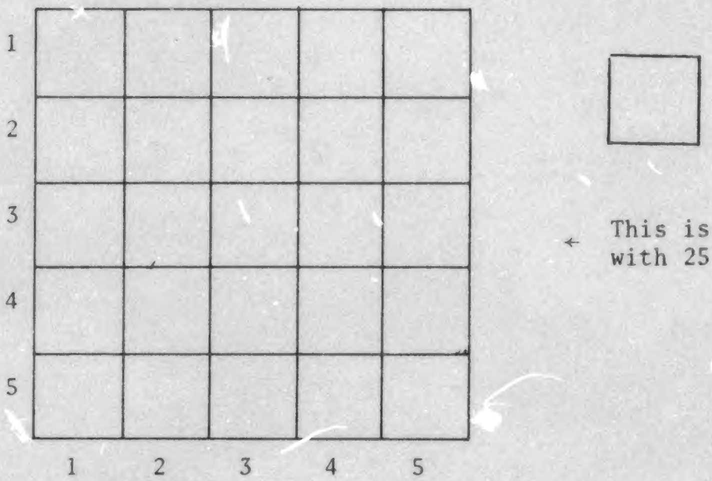
What does 5^2 mean? You might guess that it means multiply 5 by 2, or divide 5 by 2, or add 5 + 5. It doesn't!

5^2 means 5 squared or 5 x 5

How do we square a number?

Below we have squared the number 5. That is we made a 5 x 5 square that has 5 rows with 5 units in each row. How many square units does the square have? ______ Then squaring 5 gives a 5 x 5 square or 25 square units.

$$5^2 = 5 \times 5 = 25$$



← This is 1 square unit

← This is a 5 x 5 square with 25 square units

You have just seen that squaring a number is like making a square out of it and counting up the number of square units inside.
For example: $7^2 = ?$

Squaring 7 gives a 7 x 7 square with 49 square units.

$$7^2 = 7 \times 7 = 49$$

Here is another example: $8^2 = ?$

Squaring 8 gives an 8 x 8 square with 64 square units.
So.

$$8^2 = 8 \times 8 = \underline{\qquad}$$

As you can see, when we square a number we end up multiplying the number by itself. For example:

$2^2 = 2 \times 2$ $\qquad 8^2 = 8 \times 8$ $\qquad 3^2 = 3 \times 3$

Fill in the blanks.

(E27.) $5^2 = 5 \times \underline{\quad}$

(E28.) $7^2 = \underline{\quad} \times \underline{\quad}$

(E29.) $10^2 = \underline{\quad} \times \underline{\quad}$

(E30.) $6 \times 6 = \underline{\quad}^2$

(E31.) $9 \times 9 = \underline{\quad}^2$

(E32.) $.5 \times .5 = \underline{\quad}^2$

(E33.) $1/4 \times 1/4 = \underline{\quad}^2$

(E34.) $.10^2 = \underline{\quad} \times \underline{\quad}$

(E35.) $\frac{1}{2}^2 = \underline{\quad} \times \underline{\quad}$

Which are correct? Mark each one either right or wrong.

Example

$7^2 = 7 \times 7$ Right

(E36.) $3^2 = 9$ ______

(E37.) $3^2 = 6$ ______

(E38.) $17 - 4^2 = 17 - 8$ ______

(E39.) $2 \times 6^2 = 2 \times 12$ ______

(E40.) $6^2 = 6 + 6$ ______

Taking a Square Root

The opposite of squaring a number is to take the square root of a number. Below is an example of how to write a square root.

The square root of 9 would be written $\sqrt{9}$

To find the square root of 9, you find the number that would equal 9 if it were squared. What number would that be? Let's try some numbers.

Does $1^2 = 9$? No Does $2^2 = 9$? No Does $3^2 = 9$ Yes

Since $3^2 = 9$, we know that $\sqrt{9} = 3$

There is a way to find the square root of any number, but it is complicated and few people use it. It's easier to use math tables. You should, however, know how to find simple square roots that have whole numbers for an answer.

For example: What is the square root of 49?

We can try squaring some numbers and see if we get 49.

Try 5 $5^2 = 25$ (25 is too small)

Try 8 $8^2 =$ ___ (64 is too big)

Try 7 $7^2 =$ ___ (49 is the number we want)

Since $7^2 = 49$, the square root of 49 = 7

We would write this as follows: $\sqrt{49} = 7$

For each of the following problems there are four answers giver. Circle the correct answer.

Example:

$8^2 =$ 24 36 [✓ 64] 81

(E41.) $5^2 =$ 32 25 9 100

(E42.) $2^2 =$ 4 9 5 16

(E43.) $9^2 =$ 64 100 72 81

(E44.) $\sqrt{4} =$ 5 7 2 4

(E45.) $\sqrt{9} =$ 6 3 2 5

(E46.) $\sqrt{25} =$ 8 3 2 5

(E47.) $\sqrt{100} =$ 10 7 9 5

The Word "Of"

Another math symbol that is commonly used but much misunderstood is the word of. What does of mean when it's in a math problem?

"OF" MEANS "TIMES"

Whenever you see the word "of" you multiply. This is very important to remember.

If I say: take 1/5 of 40

I mean: multiply 1/5 times 40

Fill in the blanks for the statements below changing of to times.

If I say: take 5% of 30

I mean: multiply .05 ___ 30

If I say: how much is 1/4 of 100

I mean: 1/4 ___ 100

Sometimes people think that of means divide. They say, "1/4 of 8 means I divide by 8 by 4." This is true BUT: the reason you divided by 4 is that you are MULTIPLYING by 1/4. Here is how it works:

1/4 of 8 means 1/4 x 8 which is 8 ÷ 4

1/4 of 8 does not mean 8 ÷ 1/4

Finish these problems by filling in the blanks.

(E48.) 1/3 of 7 means 1/3 x __ (E49.) 5% of 20 means .05 _ 20

(E50.) 1/2 of 10 means __ x __

Answer yes or no.

(E51.) Does 1/6 of 12 mean 12 ÷ 1/6? ____

(E52.) Does 1/5 of 15 mean 15 x 1/5? ___

(E53.) Does 1/3 x 9 mean 1/3 of 9? ___

(E54.) Does 8 x 1/2 mean 1/2 of 8? ___

(E55.) Does 3% of 5 mean 5 ÷ 3%? ___

LESSON F-I

FORMULAS

Using Formulas

We use formulas when we know one thing but want to find out something else. For example:

> We have measured the length of a wall and found it to be 13 feet. We want to know how many yards long the wall is. A formula for changing feet into yards is:

$y = f/3$ where: f = feet
y = yards

You would solve the problem like this:

Since $y = f/3$ and $f = 13$ ft.

$$y = 13/3 = 3\overline{)13} = 4\ 1/3 \text{ yd.}$$

To solve the problem we first replaced the letter f with 13, then we divided the 13 by __ and got an answer of ____ yards.

Replacing Symbols with Values

A formula is like a mathematical problem except it is written with symbols instead of number values. To solve a problem with a formula you have to know how to replace the symbols with numbers. For example:

If:	and:	Then:
$A = 3 + k$	$k = 4$	$A = 3 + 4 = \underline{7}$

When a letter and a number are written together in a formula it means multiply. For example:

5n means $5 \cdot n$ or 5 times n

If: $n = 4$ Then: $5n = 5 \times 4$

What do each of the following examples mean?

(F1.) 3n means ___ times n (F2.) 2b means 2 times ___

Squaring Symbols

When a letter is squared (y^2) you multiply it times itself. For example:

If: $y = 5$ Then: $y^2 = ?$

$y^2 = y \cdot y$ So: $y^2 = 5 \times 5 = 25$

Work each of the following problems as in the example above.

(F3.) If: $n = 3$ Then: $n^2 = ?$

$n^2 = n \cdot n$ So: $n^2 = 3 \times 3 =$ ____

(F4.) If: $z = 4$ Then: $z^2 = ?$

$z^2 = z \cdot z$ So: $z^2 =$ ___ x ___ = _____

(F5.) If: $y = 2$ Then: $y^2 = ?$

$y^2 = y \cdot y$ So: y^2 = ___ x ___ = _____

(F6.) If: $b = 3$ Then: $b^2 = ?$

b^2 = ___ · ___ = _____

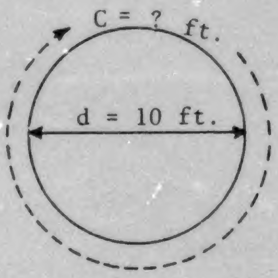
Placing Values into the Formula

A symbol that is often used in formulas where circles are involved is π. The symbol π always is given the same value. It is equal to 3.14 when written to two decimal places. For example:

The distance around a circle can be found with the formula: $C = \pi d$

Since $d = 10$ feet and $\pi = 3.14$

$C = 3.14 \times 10$ ft. = 31.40 ft.



Once you know what value each symbol has, you can replace all the symbols in your formula with a number. For example:

The formula for finding the area of a circle is: $A = \pi r^2$

If $\pi = 3.14$, and $r = 3$ feet, then, what is A?

$A = \pi r^2$ can be changed to: $A = \pi \cdot r \cdot r$

or: $A = 3.14 \times 3 \times 3$

Note: r must be squared **before** it is multiplied by π

For each of the following problems, rewrite the formulas replacing symbols with number values. (Do not work the problems)

Example: If: $Z = ky^2$ where $k = 3$ and $y = 5$

Then: $Z = k \cdot y \cdot y$

or: $Z = 3 \times 5 \times 5$

Note: y must be squared before it is multiplied by k

(F7.) If: $y = a + b$ where $a = 7$ and $b = 8$

Then: $y =$ ___ + ___

(F8.) If: $D = 3\pi c$ where $\pi = 3.14$ and $c = 9$

Then: $D = 3 \cdot \pi \cdot c$

or: $D = 3 \times$ ___ x ___

(F9.) If: $E = mc^2$ where $m = 1.2$ and $c = 1/2$

Then: $E = m \cdot c \cdot c$

or: $E =$ __ x __ x __

(F10.) If: $J = \frac{K^2}{\pi}$ where $K = 5$ and $\pi = 3.14$

Then: $J =$ ____ (Replace the symbols with numbers)

Solving Formulas in Steps

There are three steps in solving a formula. For example:

> Use the formula $M = g\overline{)n}$ to find how many miles you can drive on each gallon of gas if you can drive 160 miles on 8 gallons of gas.

M = miles per gallon,
n = number of miles driven,
g = number of gallons of gas used.

Here are the three steps:

1. Replace all symbols with the values given in the problem above.

   n = number of miles driven = ____ miles.

   g = number of gallons of gas used = ____ gallons.

   M = miles per gallon (this is what we want to find).

2. Plug the numbers into the formula in the place of the symbols.

   $M = g\overline{)n}$ so: $M = \underline{\ \ }\overline{)\underline{\ \ \ \ }}$

3. Work the problem.

   $M = 8\overline{)160} =$ _____

Our answer is that our car gets 20 miles per gallon, so it will go 20 miles on one gallon of gas.

Let's try another example of working a formula in three steps. A diagram of a fuel tank is shown below. We want to know how many cubic feet of fuel it will hold.

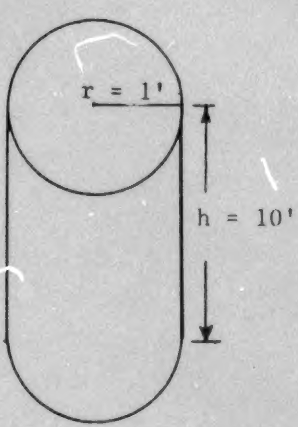
Our formula is: $v = \pi r^2 h$

where: v = cubic feet

π = 3.14

r = distance from the center to the edge of the circular end

h = height



Step 1. Replace symbols with given values

π = ______ (see box)

$r^2 = r \cdot r$ = ______ x ______ or 1 (see diagram)

h = ______ (see diagram)

Step 2. Plug the numbers into the formula in place of the symbols.

$$v = \pi r^2 h = \pi \cdot r^2 \cdot h$$

or: ______ x ______ x ______

Step 3. Work the problem

v = 3.14 x $\underline{1 \text{ x } 10}$ (multiplying 1 x 10 gives 10)

v = 3.14 x 10

v = ______ Answer

(Notice that when we have several numbers to multiply we just take two of them at a time.)

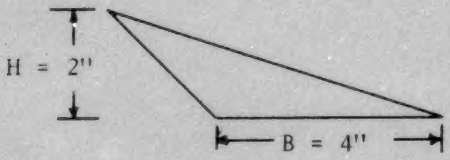
Complete the following problems by replacing letters with values. Plug them into the formula and work the problem.

(F11.) The formula for finding current in an electrical circuit is:

$C = \frac{V}{R}$ If: V = 10 volts and: R = 5 ohms

Then: C = ? amps $C = \frac{V}{R}$ or $\frac{10}{5}$ = ______ amps

(F12.) The formula for finding the area of a triangle is $A = \frac{1}{2}BH$



B = ______ in.

H = ______ in.

$A = \frac{1}{2}BH$ or $\frac{1}{2}$ x ______ x ______ = ______ sq. in.

(Remember, when you have three numbers to multiply together, multiply two of them at a time.)

(F13.) The formula for finding the distance around a circle (circumference) is $C = \pi d$

If $\pi = 3.14$ and d = 4 feet, then find the circumference (C)

$C = \pi d$ or ______ x ______ ft. = ______ ft.

LESSON F-II

EQUATIONS

Finding Missing Numbers

Solving an equation is like answering a question. For example:

(What number)	(is equal to)	(3 plus 5 ?)
n	$=$	$3 + 5$

To solve an equation you have to answer the question and find the missing number. For example:

$n = 3 + 5$

$n = ?$

We know that $3 + 5 =$ ______ so

$n = 8$ is our answer.

Understanding Equations

In the above example, it was easy to find our missing number because all the numbers were on one side of the equation. Equations are more difficult when the numbers are on both sides of the equation.

Easy	More Difficult
$n = 3 + 5$	$n - 5 = 3$

When it is difficult to see what number is missing, remember that the equation is like a question. To understand what is being asked try and state the question in words. For example:

$$\boxed{\begin{array}{c}3 + x = 7 \\ \text{or } 3 + \underline{?} = 7\end{array}}$$

Means: "What number do we add to 3 to get 7?"

$$\boxed{\begin{array}{c}3 + x = 7 \\ 3 + \underline{4} = 7\end{array}}$$

To check replace x with 4.

We have to add _ to 3 to get 7 so our answer is: $x = \underline{4}$

Can you see what this equation is asking?

$$\boxed{\begin{array}{c}b - 3 = 5 \\ \text{or: } \underline{?} - 3 = 5\end{array}}$$

Means: "What number with __ subtracted from it gives __?"

If we have to subtract 3 to get 5, our missing number must be 3 more than 5. $b = \underline{8}$

$$\boxed{\begin{array}{c}b - 3 = 5 \\ 8 - 3 = 5\end{array}}$$

To check replace 6 with 8.

Does 8 - 3 equal 5? _____ Then b = 8 is the right answer.

Here is another equation question.

$$\boxed{\begin{array}{c}7 = k + 4 \\ \text{or: } 7 = \underline{?} + 4\end{array}}$$

Means: "What number with __ added to it gives __?"

If we have to add 4 to get 7, our missing number must be 4 less than 7. k = __

$$\boxed{\begin{array}{c}7 = k + 4 \\ 7 = 3 + 4\end{array}}$$

To check replace k with 3.

Since 7 does equal 3 + 4, we know that 3 was the right answer.

Changing Equations to Words

Sometimes it is easier to see what question an equation is asking if you can see it in words. But how can you change an equation into words?

Here is an example of how to change an equation to a question.

$\boxed{9 - y = 3}$ Start your question by asking "What number do we subtract. . . ?"

"What number do we subtract from 9 to get _?"

Let's try changing another equation into a word question.

$\boxed{5 + z = 14}$ Start your question by asking: "What number do we add. . . ?"

"What number do we add to __ to get 14?"

In each of the following problems we are changing equations into word questions. Complete the changes.

Example: $6 + x = 10$ What number do we add to 6 to get 10?

(F14.) $b - 7 = 8$ What number do we subtract 7 from to get __?

(F15.) $5 + n = 9$ What number do we ___ to 5 to get 9 ?

(F16.) $5 - k = 1$ What number do we subtract from __ to get 1?

(F17.) $y + 3 = 11$ What number do we ___ to __ to get __?

(F18.) $9 = c - 2$ What number do we ________ ______from to get ____?

Checking Answers

Suppose you think you know the answer to an equation problem. You can always check your answer by plugging it into the space for the missing number. For example:

Given the equation:	You guess that:
9 - x = 5	x = 4

To check your answer, plug in 4 in place of x and see if it makes sense.

Check
9 - 4 = 5

Since 9 - 4 does equal 5, you know that your guess was right and that the missing number was 4.

Check each of the following answers and tell if it is right or wrong.

Examples:

(a)	k - 5 = 9 Answer: k = 14	Check: 14 - 5 = 9 Right
(b)	7 + n = 8 Answer: n = 15	Check: 7 + 15 = 8 Wrong

(F19.) 2 + b = 7 Check: 2 + 11 = 7

Answer: b = 11 _____

(F20.) 7 - c = 5 Check: 7 - _ = 5

Answer: c = 2 _____

(F21.) 6 = 1 + x Check: 6 = 1 + _

Answer: x = 5 _____

(F22.) 9 = k - 7 Check: 9 = _ - 7

Answer: k = 2 _____

Equations with Multiplication and Division Signs

So far, we have only looked at equations with addition and subtraction signs. We can also find missing numbers in equations where there are multiplication and division signs. Again, it may help to write the equation as a word question. For example:

$\boxed{3 \text{ x } n = 12}$ Means: "What number multiplied by 3 will give 12?"

Since 3 will go into 12 four times, we might guess that our missing number is $\underline{4}$.

To see if 4 really was the missing number replace the n with 4 as a check.

Check:

$3 \text{ x } n = 12$

$3 \text{ x } \underline{4} = 12$

Is 3 x 4 equal to 12? _____ Then 4 is a correct guess.

Here is an example of an equation with division.

$\boxed{6/b = 2}$ Means: "What number will divide into 6 and give __?"

(Notice that 6/b means the same as 6 ÷ b.)

What number will go into 6 two times? Three goes into 6 two times so we might guess that b is 3.

Let's check our answer

Check:

$6/b = 2$

$6/\underline{3} = 2$

Does 6/3 = 2? _____ Then 3 is the missing number.

For each of the following problems we are changing equations into word questions. Complete the changes.

Example: $a \times 3 = 18$ Means: "What number multiplied by 3 gives 18?"

(F23.) $\frac{k}{4} = 12$ Means: "What number divided by __ gives 12?"

(F24.) $2y = 8$ Means: "What number multiplied by __ gives __?"

(F25.) $24 \div n = 6$ Means: "What number divided into __ gives __?"

(F26.) $27/b = 3$ Means: "What number divided into __ gives __?"

Notice that in the above examples $\frac{k}{4}$ means k divided by 4, and 2y means 2 times y.

Checking Answers with Multiplication and Division

For each of the following problems, check the answer by plugging it into the space for the missing number. Then mark the answer right or wrong.

Example: $9 \times y = 18$ Check: $9 \times \underline{3} = 18$
$y = \underline{3}$ wrong

(F27.) $27/n = 9$ Check: $27/\underline{3} = 9$
$n = \underline{3}$ ______

(F28.) $6 \times c = 24$ Check: $6 \times \underline{\quad} = 24$
$c = \underline{6}$ ______

(F29.) $\frac{x}{7} = 14$ Check: $\frac{\underline{\quad}}{7} = 14$
$x = \underline{2}$ ______

Shifting Numbers to Simplify Equations

If you are having trouble finding what the missing number is, shift the numbers or symbols so that the missing value is alone on one side of the equation.

When you shift a number or symbol you have to make it do the opposite of what it was doing. Here are some examples:

(F30.)
Change: $n/3 = 6$
To: $n = 6 \times 3$

We were dividing by 3 so when we shifted we multiplied by __.

(F31.)
Change: $k \times 9 = 18$
To: $k = 18/9$

We were multiplying by 9 so when we shifted we divided by __.

(F32.)
Change: $8 = z + 5$
To: $8 - 5 = z$

We were adding 5 so when we shifted we ________ 5.

(F33.)
Change: $n - 3 = 11$
To: $n = 11 + 3$

We were subtracting 3 so when we shifted we ________ 3.

(F34.)
Change: $7 = 21/b$
To: $7 \times b = 21$
And: $b = 21/7$

Here we shifted twice. First to get the b out from under the 21. Second, to get the b alone, we shifted the __.

(F35.)
Change: $7 - k = 4$
To: $7 = 4 + k$
And: $7 - 4 = k$

Here if we had shifted the 7 we would get $- k = 4 - 7$. To avoid this we first shift the k. Second, to get the k alone, we shift the __.

Finish each of the following problems and check the answer. If the missing value is difficult to guess, we can shift numbers to simplify the equation.

(F36.) $x + 3 = 7$ Means: "What number added to 3 gives 7?" $x =$ ____

Check: __ $+ 3 = 7$

(F37.) $y - 2 = 6$ Means: "What number with __ subtracted from it gives 6?" $y =$ ____

Check: __ $- 2 = 6$

(F38.) $d/5 = 20$ We were dividing by 5 so we can shift and ________ by 5. $d =$ ____

$d = 20 \times 5$

Check: __$/5 = 20$

(F39.) $16 = k \times 8$ Means: "What number multiplied by __ gives __?" $k =$ ____

Check: $16 =$ __ $\times 8$

(F40.) $n - 4 = 3$

$n =$ ____

Check: __ $- 4 = 3$

(F41.) $y/5 = 15$

$y =$ ____

Check: __$/5 = 15$

(F42.) $b \times 3 = 21$

$b =$ ____

Check: __ $\times 3 = 21$

(F43.) $9 = 4 + x$

$x =$ ____

Check: $9 = 4 +$ ____

(F44.) $3/k = 1$

$k =$ ____

Check: $3/$__ $= 1$

(F45.) $8 = 2y$

$y =$ ____

Check: $8 = 2 \times$ __

LESSON G-I

PERCENTS OF NUMBERS

What do you save with 25% discount on a $20 watch?

How much is 5% of 86?

What is the cost of a 10% tax increase if your taxable income is $5,000?

The above questions are examples of percentage problems. You will run into percentage problems when you:

1. Do your income tax.
2. Try to find out how much time payments cost you.
3. Want to know how much you can save on a sale.
4. Read charts and graphs.

Working A Percentage Problem

Here is an example: 15% of 200 = ?

change: 15% of 200

to: .15 x 200

or: $\begin{array}{r} 200 \\ \underline{\times\ .15} \end{array}$

In order to do this problem, we changed the 15% to ________

and the of 200 to times 200.

Here is another example: 35% of 20 = ?

change: 35% of 20

to: .35 x 20

or: $\begin{array}{r} 20 \\ \underline{\times\ .35} \end{array}$

This time, we have changed the 35% to .35, and the of 20

to _____ 20.

RULE: To take a percent of some number, change the percent to a decimal and multiply.

Look over these examples and fill in the missing information.

13% of 40 means .13 times 40

71% of 15 means .71 times 15

52% of 35 means .52 times 35

16% of 10 means .16 times __

22% of 67 means .22 _____ 67

59% of 59 means .59 _____ __

At this point you should know how we work a percentage problem even though you may not yet see why we do it this way.

To take a percent of some number, we change the percent to a _______ and _____. (If you don't remember look back at the rule at the bottom of the first page.)

Perhaps you think it's odd that "of" means multiply instead of divide. People often get confused and think that:

15% of 20 means 20 divided by 15%

THIS IS WRONG--"OF" DOES NOT MEAN "DIVIDE"--IT MEANS "MULTIPLY!"

Taking a percent is like taking a fraction of a number.

For Fractions:

1/4 of 20 means: 1/4 times 20
Not: 20 divided by 1/4

In the same manner,

For Percents:

25% of 20 means: .25 times 20
Not: 20 divided by .25

FOR FRACTIONS OR PERCENTS-- "OF" MEANS "MULTIPLY!"

Percent of--means--percent times.

What Are Percents?

Percents are very much like fractions and decimals. They are used to talk about things that have been separated into parts.

A percent is like a fraction.

50% of $5 = $2.50

1/2 of $5 = $2.50

These two statements mean the same thing.

A percent is also like a decimal.

50% of $20 = $10

.50 x $20 = $10

These two statements mean the same thing.

How Do Y[illegible] Change a Percentage into a Fraction?

Percents mean one hundredths.

20% means 20/100

5% means 5/100

71% means 71/100

You can always change a percentage into a fraction with 100 as the bottom number.

Can you change these percentages into fractions?

(G1.) 3% = ____ (G2.) 17% = ____ (G3.) 33% = ____ (G4.) 99% = ____

How do You Change a Percentage into a Decimal?

You have already learned how to change a percentage into a fraction with 100 as the bottom number. (Fill in the missing number)

20% = 20/100

15% = ___/100

To change a percent to a decimal, first change it to a fraction, then change the fraction to a decimal.

For example, change 21% to a decimal.

1st change the percent to a fraction. 21% = 21/100

2nd change the fraction to a decimal. 21/100 =

100/$\overline{21}$ = .21

Change each of the following percentages 1st to a fraction and 2nd to a decimal. (Fill in the blanks.)

Example: 5% = 5/100 = .05

(G5.) 9% = 9/100 = _____

(G6.) 12% = 12/100 = _____

(G7.) 20% = ___/100 = _____

(G8.) 7% = _____ = _____

(G9.) 35% = _____ = _____

Percents to decimals by moving the point.

A fast way to divide by 100.

You may have noticed that dividing by 100 gives the same answer as moving your decimal point two places to the left.

$$\frac{38}{100} = .38$$

$$\frac{1}{100} = .01$$

$$\frac{169}{100} = 1.69$$

In changing a percent to a decimal we always divide by 100

$$\underline{8}\% = \frac{8}{100} \text{ or } .\underline{08}$$

$$\underline{17}\% = \frac{17}{100} \text{ or } .\underline{17}$$

$$\underline{254}\% = \frac{254}{100} \text{ or } 2.\underline{54}$$

Since you always divide by 100 when you change a percent to a decimal, you can always do it by moving the decimal point two places to the left. This is much faster than actually dividing by 100. For example:

$$25\% = .25 \qquad 7\% = .07 \qquad 500\% = 5.00$$

Changing a Fraction or Decimal to a Percent

Decimals - Percents

We have already seen that we can change a percent to a decimal by moving the decimal point over two places to the left.

To change a decimal to a percent we just do the opposite and move the decimal point over two places to the right.

Percent to Decimal	Decimal to Percent
35% = .35	.65 = 65%

Droping and Adding Zeros

When we change a decimal to a percent, we may have to add or drop zeros in our answer. For example:

.05 = 05% or: 5% (Don't need the zero in front)

.7 = 70% (Had to add a zero to move the decimal two places)

Don't make the mistake of saying .7 = 7%

Remember, .07 = 7%, but .7 is the same as .70 or 70%

Change each of the following decimals to percents by moving the decimal point two places to the right.

Examples: .16 = 16% .05 = 5% .3 = 30%

(G10.) .51 = ____% (G11.) .07 = ___% (G12.) .2 = ____%

(G13.) .01% = ____% (G14.) .99 = ___% (G15.) 7.5 = ___%

Fractions - Percents

To change a fraction into a percent, first divide the bottom into the top to get a decimal. Then change the decimal into a percent. For example:

$$4/5 = 5\overline{)\,4.00} = .80 \qquad .80 = 80\%$$

Change each of the following fractions to percents.

(G16.) 1/4 = ?%

$$\begin{array}{r} .25 \\ 4\overline{)\,1.00} \end{array}$$

.25 = __%

(G17.) 1/2 = ?%

$$\begin{array}{r} .50 \\ 2\overline{)\,1.00} \end{array}$$

.__ = __%

(G18.) 3/4 = ?%

$$\begin{array}{r} . \\ 4\overline{)\,3.} \end{array}$$

.__ = __%

Taking More than 100%

What happens when you have more than 100%? We said before that 100% means the whole thing. If we were talking about a pie, then 100% would be the whole pie. But what if you started with one pie and ended up with two pies? You would say that you had twice as much as you started with or 200% of what you started with.

For example:

200% of 1 pie = 2 pies

If you have more than 100%, then you end up with more than you started with.

Change 150% to a decimal.

$$150\% = \frac{150}{100} = 1.50$$

You see that when we have more than 100% it changes to a decimal that is greater than 1.

Change these percents first into fractions, and then into decimals:

Example: $\boxed{300\% = \frac{300}{100} = 3.00}$

(G19.) 250% = $\frac{\quad}{100}$ = ___.___ (G20.) 165% = $\frac{\quad}{100}$ = ___.___

(G21.) 400% = $\frac{\quad}{100}$ = ___.___

Work the following percentage problems.

(G22.) 200% of 5 = 2.00 x ___ = _____

(G23.) 150% of 8 = ___.___ x 8 = _____

(G24.) 100% of 23 = ___.___ x _____ = _____

When you have a percent that is an even hundred you can change the percent directly to a whole number. For example:

$\boxed{\underline{200\%} \text{ of } 5 = \underline{2} \times 5 = \underline{10}}$ (Because 200% = 2.00 or 2)

Work the following problems changing each percent into a whole number.

(G25.) 100% of 5 = __ x 5 = _____

(G26.) 300% of 5 = __ x 5 = _____

(G27.) 500% of 5 = __ x 5 = _____

(G28.) 800% of 5 = __ x 5 = _____

LESSON G-II

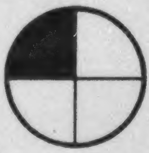
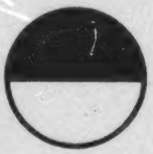
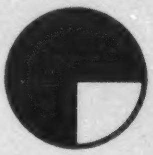
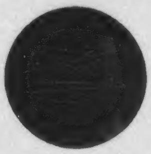
ADDING AND SUBTRACTING PERCENTS

What Do We Mean by 100%?

If I told you you were 100% correct on your test it would mean that you got the whole test right.

100% = The whole thing

Sometimes it's useful to make a percentage diagram. If I draw a circle, I can say that the whole circle is 100%. I can then divide the circle into parts and talk about these parts as percents of the whole circle.

What part of the circle is black?				
Percent	25%	50%	75%	100%
Fraction	1/4	1/2	3/4	1

Notice that I can describe a part of the circle either as a percent or as a fraction. I can say either:

1/4 of the circle is black

or

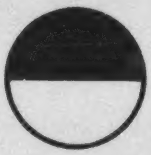
25% of the circle is black

Both mean the same thing.

Study the above diagram carefully to see how percents can be expressed as fractions or as parts of a circle. Then try the following problems.

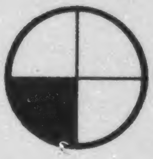
Mark each item, either True or False. An item is true if the percent is equal to the fractional value. (For the circles, compare the black portion.)

(G29.) 25% =



____

(G30.) 50% =



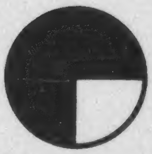
____

(G31.) 3/4 = 75% __________

(G32.) 25% = 1/4 __________

(G33.) 100% = 1 __________

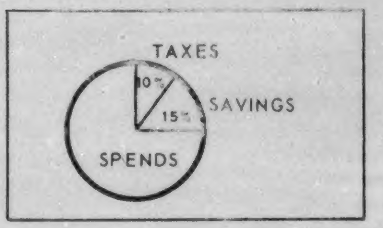
(34.) 75% =



____

Percentages and Pie Graphs

Finding a missing percentage.



Percentages are often used in graphs to represent breakdowns of earnings, spendings, budgets, etc. This circle represents an individual sailor's pay and how he spends it.

Notice that this graph tells nothing about how many dollars are earned or spent. All that it tells is what part of his total wages was saved, spent, or went for taxes.

We did not write in what percent of his wages he spends. Can you tell what percent he spends by looking at the pie graph? The answer is "yes you can." Here is how you can do it:

The key to solving this type of problem is based on two facts.

1. The whole thing equals 100%.
2. The parts must add up to 100%.

If 100% is his whole pay check and he takes out 10% for taxes and 15% for savings, then how much is left to spend?

He spends: 100% - (10% + 15%)
or: 100% - 25%

Since 100% - 25% = ____% he must spend 75% of his money.

(Note that we added the 10% and 15% together to get 25%.)

Can we add percents together?

You can add and subtract percentages just like you would add and subtract other numbers.

Since we can add and subtract percentages, we can say that:

	His whole salary		What is taken out		What he has to spend
		-		=	
or:	100%	-	25%	=	75%

Finding the Value of a Percentage

If we know a man spends 75% of his money, do we know how much money he spends? No! We know what part of his money he spends but we can't tell how much. That is because we don't know how much he makes. But if we know how much he makes, then we can tell what he spends.

Let's say that this sailor makes $5,000 a year. How much would he spend if 10% goes for taxes, 15% for savings, and the rest (75%) he spends? Each amount is some percent of his earnings ($5,000).

(G35.) Taxes = 10% of $5,000 = $__________

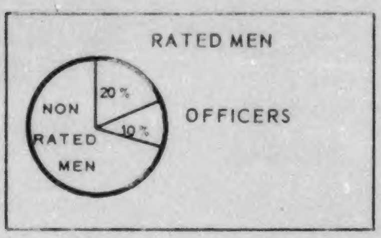
(G36.) Savings = 15% of $5,000 = $__________

(G37.) Spendings = 75% of $5,000 = $__________

(G38.) Total 100% of $5,000 = $__________

Here is another example of a pie graph percentage problem.

The pie graph below shows what percentage of the men in the Navy are rated men and officers. First find what percentage of the men are rated, then find how many men are non-rated.



Total number of

men = 500 thousand.

What must we know in order to find out how many Navy men are non-rated? We need to know what percent are non-rated. How can we find out what percent are non-rated?

We know that the parts of the circle must add up to 100%. So:

Officers + Rated men + Non-rated men = All Navy men.

or

10% + 20% + ? % = 100%

We need to find the missing percent.

Since all Navy men together equal 100%, the percentage of non-rated men can be found by subtracting 30% (rated men + officers) from 100%.

So non-rated men = 100% - 30% = ____%

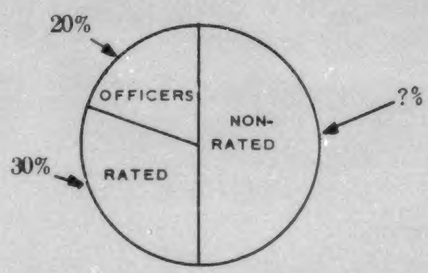
At this point, we have found that 70% of the men are non-rated. How do we now find out how many men are non-rated? We do it by forming a percentage problem. There were 500 thousand men in all so:

70% of 500 thousand men are non-rated

and: 70% of 500 = __________

Our answer is that we have 350 thousand non-rated men.

Try this problem.



(G39.) If we have 400 thousand men and

20% are officers

30% are rated

the rest are non-rated

Then how many non-rated men do we have?

% non-rated + 20% + 30% = 100%

so: __% are non-rated men.

__% of 400 thousand = ________ thousand non-rated men.

Combining Multiple Percentages

Sometimes a percentage problem is given where you have to take several percentages of the same amount.

10% of $20 + 30% of $20 + 15% of $20 = ?

There are two ways to do this problem.

The slow way	The fast way
10% of $20 = $2	10% + 30% + 15% = 55%
30% of $20 = $6	55% of $20 = $11
15% of $20 = $3	
so	
$2 + $6 + $3 = $11	

It's faster to combine percents of the same amount. In this case each percentage is taken for $20 so they can be combined.

Here is another example where you can add the percents to save time. In an officer's candidate school, 5,000 men applied for admission. 10% were dropped out for physical disabilities, 15% were dropped because they couldn't pass the examinations, and 25% changed their minds about becoming an officer during training. How many were dropped in all?

The slow way

10% of 5,000 = ____

15% of 5,000 = ____

25% of 5,000 = ____

so

500 + 750 + 1250 = ____ Answer

The fast way

10% + 15% + 25% = ____%

50% of 5000 = ________ Answer

(Both answers should be the same if you work the problems correctly)

LESSON G-III

PERCENTAGE PROBLEMS

Percentage and Borrowing

Suppose you are buying a car and are borrowing the money to pay for it. You will be charged interest on the loan. How much you end up paying will depend on the rate of interest and how long you borrow the money for. When you buy on time, be sure you know how much it is costing you.

What is the rate of interest?

	Loan Sharks, Inc.	Community Bank
	Interest = 1% a month	Interest = 7% a year
For 1 year	You pay $12 for each $100 you borrow for 1 year.	You pay $7 for each $100 you borrow for 1 year.

You might think that 1% a month is a lower cost loan than 7% a year. If we look at the rate of interest we see differently. Rate of interest is the percent of interest paid each year. A charge of 1% per month is 12% per year.

You pay the bank 7% a year in interest. You pay Loan Sharks, Inc. 1% a month which is 12% a year.

Who really has the higher rate? ____________

To compare the cost of two loans, you must know what rate of interest they charge. A rate of interest, tells you what percent you pay to borrow the money for a year.

What if you borrow money for more or less than a year?

Suppose you borrow $100 at a rate of 8% and pay it back in 6 months. It should cost you less than if you had borrowed the money for a full year.

Principal = $100
Rate = 8%
Time = 1/2 year
Interest = ?

The loan would cost 8% of $100 or $8 for each year you borrow.

If you borrow the money for 1/2 year it will only cost you 1/2 of $____, 1/2 x $8 = $____. The loan will cost $4 for 1/2 year.

What would it cost to borrow $50 for 2 1/3 years at a rate of 6%?

Principal = $50
Time = 2 1/3 years
Rate = 6%
Interest = ?

To borrow $50 for 1 year it would cost 6% of $50 = $3.00.

To borrow $50 for 2 1/3 years it would cost _____ times $3.00 = $7.00.

Here is a rule you can use to find the cost of interest.

To find out how much a loan costs, multiply how much it costs for 1 year by the number of years you borrow the money.

Work the following problem to find the cost of interest on the loan.

(G40.)

Principal = $200

Rate = 5% Cost for 1 year = 5% of $200 = $____

Time = 9 months Cost for 9/12 of 1 year = 9/12 x $10 = $____

Percentage and Savings

When you borrow money you pay interest. When you save money, on the other hand, the bank pays you interest.

For example, how much interest would you get if you put $5,000 in a savings account for 5 years at a rate of 5%?

Principal = $5,000

Rate = 5%

Time = 5 years

Interest for 1 year = 5% of $5,000 = $250.

Interest for 5 years at $250 a year = 5 x $____ = $1250

(G41.) How much interest would you earn if you left your $5,000 for 6 months in a bank that pays an interest rate of 6%?

Principle = $______

Rate = ____%

Time = ____ years

Interest for 1 year = .06 x $5,000 = $_____.

Interest for 1/2 year = 1/2 of $____ = $____

(G42.) How much interest would you earn if you left your money in the same bank for 2 1/3 years?

Interest for 1 year = $300 (same as above).

Interest for 2 1/3 years = ____ x $300 = $____

Percentages and Taxes

Every year around February every working man receives a statement of how much he has earned (a W-2 Form). He then has to pay a tax on his earnings. You have to give back to the government a percentage of what you earn. The more you earn, the larger the percentage that you pay back. (They never really ask for 100%, it only seems that way.)

For example, if you earn $6,000 a year and have to pay 10% back in taxes, how much would you pay.

10% of $6,000
is .10 x $6,000 = $600

Here is another example.

If you made $7,000 and had to pay back 10% in taxes, how much would that be?

10% of $7,000 = ___.___ x $7,000 = $700

Now try these problems.

(G43.) 20% of $800 = __________

(G44.) 45% of $1,000 = __________

(G45.) 9% of $200 = __________

(G46.) 1% of $45 = __________

(G47.) You make $8,000 in taxable income each year, and you pay 15% of this amount in taxes. How much do you pay?

15% of $8,000 = __________

(G48.) Your taxable income for last year was $5,500. You paid back 10% of this in taxes. Your tax was ____% of $_____ = $_____.

(G49.) Your state has a 3% sales tax. If you paid this sales tax on $2,500 of the money you spent last year, how much tax did you pay?

__% of $_____ = $_____

Percentage and Profits

In the Navy, the Storekeepers use percentages in deciding how much to charge for the things that they sell you. You have to pay the amount it cost them plus a percentage of the cost to cover expenses. Since they are not out to make a profit, and (when out at sea) you don't pay taxes, you can buy things much cheaper.

How much do you save?

How much less would it cost to buy a camera that wholesales for $20 at the ship's store? (Fill in the blanks.)

		Type of Expense	Amount
Ship's Store		Store's cost	$20.00
	+	15% overhead = 15% of $20 =	+$ 3.00
		You pay	$23.00

		Type of Expense	Amount
Civilian Store		Store's cost	$20.00
	+	15% overhead = 15% of $20 =	$_____
	+	35% profit = 35% of $20 =	+ $_____
		You pay	$30.00

How much do you save at ship's store? $30.00 - $23.00 = $__.__

Commissions

Salesmen sometimes are paid on a commission basis. This means that they are paid a percent of what they sell.

If a car salesman sells a car for $2500 and his rate of commission is 8%, how much does he make? (Fill in the blanks.)

Price = $2,500

Rate = ___%

Commission = ___

Commission = ___% of $_____ = $200

Work the following problems.

(G50.) If a ship's store adds a 10% markup, how much will they charge for a watch that cost them $50?

Store pays: $50
Markup: $___ (___% of $50 = $___)
Store charges: $___

(G51.) In California a 5% sales tax is added on to most things you buy. If you buy a stereo for $200 and sales tax = 5%, how much do you pay in all?

Price: $200
Tax: $___ (___% of $____ = $___)
Total: $____

(G52.) A salesman gets a 20% commission. If he sells a car for $2,000 what is his commission?

Commission = $____ (____% of $____ = $____)

Discounts

Sometimes items are sold at discount prices because they are hard to move. What does a store mean when it says that all items are being discounted by 20%? It means they are subtracting 20% off the original price.

What would it cost you to buy a $10 watch being sold at a 20% discount?

20% of $10 = $____

$10 - $_____ = $ 8

In this case with the 20% discount you save $2. The discount price is $8.

As you work the following problems- watch out! Be sure and pay attention to whether they are asking how much you pay or how much you save.

(G53.) How much do you pay for a $4 shirt that has been discounted 25%?

You save: __% of $____ = $____

You pay: $4 - $____ = $____

(G54.) How much do you save on a $90 suit that has been discounted 19%?

You save __% of $____ = $____

(G55.) Ship's store has a camera priced at $30. The Cut-Rite camera store has the same camera which it normally sells for $40 at a discount of 15%. What is the difference in prices?

Ship's Store Price = $____

Cut-Rite Price = $40 - $____ or: $______

Price Difference = Cut-Rite Price - Ship Store's Price.

Price Difference = $______

LESSON G-IV

PERCENTAGE EQUATIONS

The Parts of a Percentage Problem

There are three parts to a percentage problem, the amount you start with, the percent you take, the amount you end up with. For example:

1. You start with $30.
2. You take 20%.
3. You end up with $6.

20% of $30 = $6

Up to now, you have been given the first two parts and asked to find out what you end up with. However, you could be given any two parts and asked to find the third. For example:

(a)	(b)	(c)
20% of 30 = ?	20% of ? = 6	?% of 30 = 6

From Problems

Below, each of these problems has been written as an equation.

(a)	(b)	(c)
$.20 \times 30 = n$	$.20 \times n = 6$	$n \times 30 = 6$

To Equations

Notice that to change the problems into equations we changed the word ___ to x and changed 20% to the decimal .20. We used the letter n to stand for the value we were trying to find.

Any percentage problem you may be given can be set up in one of these three ways. In this lesson we will consider how to set up and solve these three types of percentage equations.

Finding a Missing Percentage

We have covered problems where you find a percentage of a number.

20% of 30 = ?

.20 x 30 = 6.00

To work this problem we change the percent to a decimal and of to times, then we multiply.

Another type of percentage problem is where you have to find a missing percentage:

?% of 30 = 6

To work this type of problem we first write it as an equation:

$n \times 30 = 6$

To find n we shift the 30 and divide by it:

$n = 30\overline{)\,6}$

When we divide 30 into 6 we will get a decimal answer that we can then change into a percent.

$$\begin{array}{r} .20 \\ 30\overline{)\,6.00} \end{array} = ?\%$$

Since .20 = __%, our answer is 20% of 30 is equal to 6.

Here is another example of a problem where the missing number is a percent. ?% of 8 = 2

First we make an equation: $n \times 8 = 2$

Next we shift the 8 and divide: $n = 8/\overline{\;2\;}$

We will get a decimal answer that can be changed to a percent.

$$8/\overline{\;2\overset{\cdot}{.}00\;} \qquad .25 = \underline{\quad}\%$$

Our answer is: $\underline{25}$% of 8 = 2. You can check this answer by taking 25% of 8 and finding out if it does equal 2.

25% of 8 = .25 x 8 = _____ (our answer checks)

Don't make this common mistake.

When finding a missing percentage, people sometimes make the mistake of dividing by the wrong number. For example: ?% of 10 = 4

$$\boxed{\begin{array}{l} n \times 10 = 4 \\ n = 4/\overline{\;10\;} \end{array}} \leftarrow \text{This is wrong}$$

Asking what percent of 10 is equal to 4, is like asking what number multiplied times 10 is equal to 4. In order to get 4 for an answer, we have to multiply 10 by a decimal number that is less than 1. $4/\overline{\;10\;}$ would give an answer larger than 1.

$$\boxed{\begin{array}{l} n \times 10 = 4 \\ n = 10/\overline{\;4\;} \end{array}} \leftarrow \text{This is right}$$

Mark each of the following problems right or wrong to indicate if they are being worked correctly. (Do not work the problem)

Examples:

?% of 6 = 3
n x 6 = 3
n = 3/ 6.00 Wrong

?% of 12 = 3
n x 12 = 3
n = 12/ 3.00 Right

(G56.) ?% of 10 = 2
n x 10 = 2
n = 10/ 2.00 _____

(G57.) ?% of 16 = 4
n x 16 = 4
n = 4/ 16.00 _____

(G58.) ?% of 30 = 3
n x 30 = 3
n = 3/ 30.00 _____

(G59.) ?% of 60 = 5
n x 60 = 5
n = 60/ 5.00 _____

Complete each of the problems on page 27 to find the missing percentages.

Example:

?% of 25 = 5
n x 25 = 5
n = 25/ 5.00 = .20 or 20%
20% of 25 = 5

(G60.) ?% of 100 = 26

n x 100 = 26

n = 100/ __ = .26 or 26%

___% of 100 = 26

(G61.) ?% of 50 = 8

n x __ = 8

n = __/ 8.00 = .16 or ___%

___% of 50 = 8

(G62.) ?% of 38 = 19

n x __ = 19

n = 38/ 19.00 = .50 or ___%

___% of 38 = 19

(G63.) ?% of 16 = 4

n x __ = __

n = __/ 4.00 = .__ or ___%

___% of 16 = 4

(G64.) ?% of 10 = 6

_ x _ = _

n = _/ _.__ = .___ or ___%

___% of 10 = 6

(G65.) ?% of 20 = 6

_ x _ = _

n = _/ _.__ = .__ or ___%

___% of 20 = 6

Finding What You Are Taking a Percentage Of

So far we have covered two kinds of percentage problems:

Finding a percent of a number: 5% of 80 = ?

Finding a missing percentage: ?% of 80 = 4

The third type of percentage problem we may run into is:

Finding what number we are taking a percentage of: 5% of ? = 4

Again, we have to turn the problem into an equation and shift numbers.

Here is an example of a problem where we are trying to find the number that we are taking a percentage of: 3% of ? = 6

First we make an equation: $.03 \times n = 6$

Next we shift the .03 $n = .03/\overline{6}$

Now all that remains to be done is work the problem.

$$.03/\overline{6.00} \quad = \quad 3/\overline{600} \quad = \quad \_\_\_\_$$

Our answer is 200 so we can say: 3% of 200 = 6

(You can check this answer by taking 3% of 200 and seeing if you do get 6)

Check:

$$3\% \text{ of } 200 \quad = \quad .03 \times 200 \quad = \quad \_\_\_\_$$

Here is another example of a problem where we are trying to find the number that we are taking a percentage of: 2% of ? = 5

First we make an equation: $.02 \times n = \_$

We shift the _ and change from multiplication to division: $n = .\_\_/\overline{5}$

We finish the problem by dividing: $n = .02/\overline{5.00} = 2/\overset{250}{\overline{500}}$

Our missing value, then, was 250.

Putting the missing value in the equation we have: 2% of ___ = 5

(You can check this answer by taking 2% of 250 to see if it does equal 5)

Work each of the following problems to find the number we are taking a percentage of.

Example:
7% of? = 14

.07 x n = 14

n = .07/ 14

$.07\overline{)14.00} = 7\overline{)1400}$ with 200 above

Answer = 200

(G66.) 15% of ? = 3

.15 x n = 3

n = .___/ 3

n = .15/ 3.00 = 15/ ___

Answer 15% of ___ = 3

(G67.) 25% of ? = 7

.___ x n = 7

n = .25/ ___

n = .25/ 7.00 = ___/ ___

Answer 25% of ___ = 7

(G68.) 4% of ? = 2

.___ x ___ = ___

n = ___/ ___

n = .04/ ___ = 4/ ___

Answer 4% of ___ = 2

(G69.) 5% of ? = 1

.___ x ___ = 1

n = ___/ ___

n = ___/ ___

Answer 5% of ___ = 1

(G70.) 50% of ? = 4

.___ x ___ = 4

n = ___________

n = ___/ ___

Answer 50% of ___ = 4

Changing Word Problems to Equations

A percentage question is often given as a word problem. Before you can work the problem, you have to change it into an equation.

For any percentage problem there are always three basic parts:

1. The amount you start with.
2. The percentage you take (rate of percent).
3. The amount you end up with.

Your equation can always be set up in this form:

__% of __ = __

You will be given two of the values and will have to find the third.

Here is an example of a word problem changed into an equation.

A tire shop has discounted all their tires 25%. They tell you that you can save $5 on a tire for your car. What was the price of the tire before they marked it down?

First we set up the problem:	__% of __ = __
Next we plug in the given values:	$\underline{25}$% of $\underline{?}$ = $5
Then we change the problem into an equation:	$.\underline{25} \times n = 5$

Now all that remains to be done is to find the missing value.

$.25 \times n = 5$

$n = .\underline{25}\overline{)\underline{\quad\quad}} = 25\overline{)\underline{\quad\quad}}$ Answer = 20

The price of the tire was $__.

Change the following word problems into equations. (Do not work the problems)

Example: A bank charges 12% rate of interest. If you borrow $500 what will you have to pay each year in interest?

12% of $500 = ?

.12 x 500 = n

(G71.) A store has marked a $8 shirt down $2. By what percent did they discount the shirt?

___% of $___ = $___

n x ___ = ___

(G72.) What is the principal of a loan (the amount borrowed) if the rate of interest is 6% and you must pay $20 each year in interest?

___% of ___ = ___

___ x ___ = ___

(G73.) This year your savings bank paid you $12 in interest. The principal (the amount you put in the bank) was $240. What rate of interest does this bank pay?

___% of ___ = ___

___ x ___ = ___

(G74.) A car salesman makes a 10% commission on each car he sells. If he sells a car for $3,200 how much is his commission?

___% of _____ = _____

___ x _____ = _____

(G75.) Principal = $400

Interest = $48

Rate = ?

___% of ____ = ____

___ x ____ = ____

Following are examples of the three types of percentage problems. In one you are asked to solve for a missing percent, in another, for the amount you are taking a percent of, in the third, for the amount that we get when we take the percent.

Complete each problem. First change the problem to the form ?% of ? = ? then, write the problem as an equation and find the missing value.

(G76.) Principal = $500   __% of ____ = ____

Rate = 8%   .__ x ____ = n

Time = 10 years   **Interest = $____ per year**

Interest = ?   Interest for 10 years is:

10 x $__ = $____ (Answer)

(Note that when time is involved we first find the cost of interest for one year, then multiply by the number of years.)

(G77.) **If the principal of a loan (the amount borrowed) is $600 and the annual interest charge is $36, what is the rate** of interest?

__% of ____ = ____

__ x ____ = ____

Rate = ____ %

(G78.) If a salesman sells houses at 6% commission and he makes a commission of $1,800 on one house, how much did he sell the house for?

__% of ______ = ______

The house sold for $________

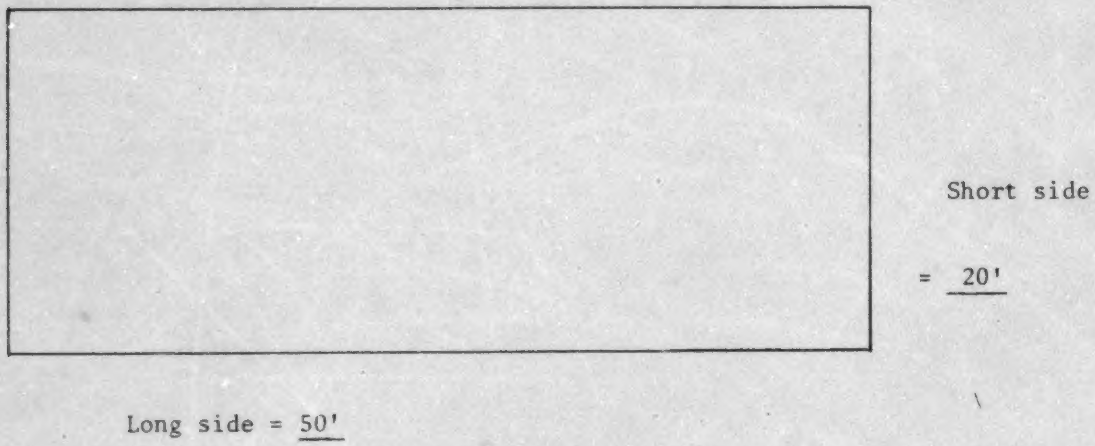
LESSON H-I

MEASURING DIMENSIONS

Perimeter--The Distance Around

If you were going to fence in a field, you would want to know how many feet of fencing you needed. You would have to find out how far it was around the edge of the field. The distance around the edge of a figure is called its perimeter.

The below figure is a drawing of a field. Your task is to find out how many feet of fencing it will take to fence it in. (20' means 20 feet)



We have four sides to the fence. If we find out how long each side is, and then add them all together, we will know how long the whole fence is.

Look at the sides: The two long sides are each ____ feet long.

The two short sides are each ____ feet long.

Add the 4 lengths. 50 feet + 50 feet + 20 feet + 20 feet = 140 feet

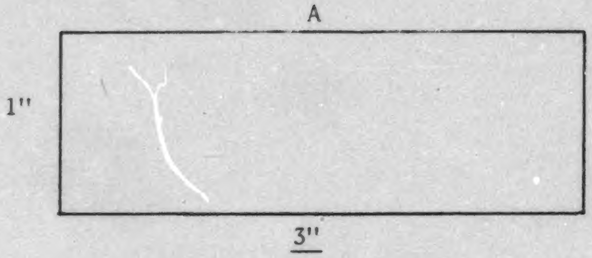
You have just measured the perimeter of a rectangle. When we measure the outside edge of anything, we say that we are measuring its perimeter.

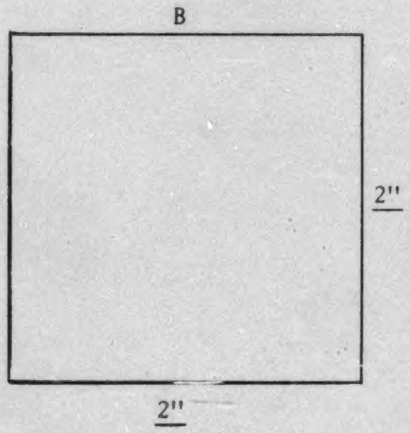
Here is another example. What is the perimeter of a rectangular garden that is 30 feet wide and 60 feet long?

____ feet + ____ feet + ____ feet + ____ feet = 180 feet

Sometimes two different shapes can have the same perimeter. Find the perimeters of the following two figures. (3" means 3 inches)

(H1.)



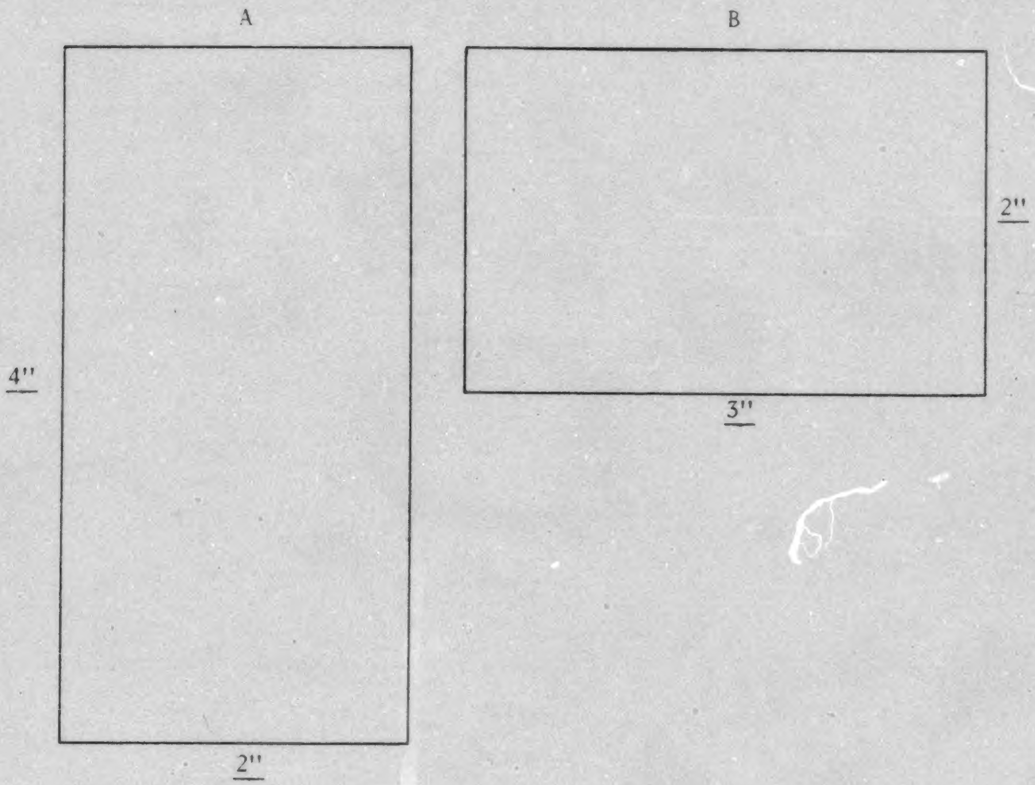


Perimeter A = ____ inches

Perimeter B = ____ inches

Are the perimeters equal? ____

(H2.) Here are two more figures. Which has the larger perimeter?



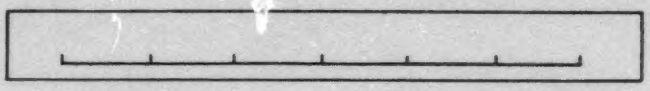
Perimeter A = ____ inches

Perimeter B = ____ inches

Perimeter ____ is ____ inches longer than perimeter ____.

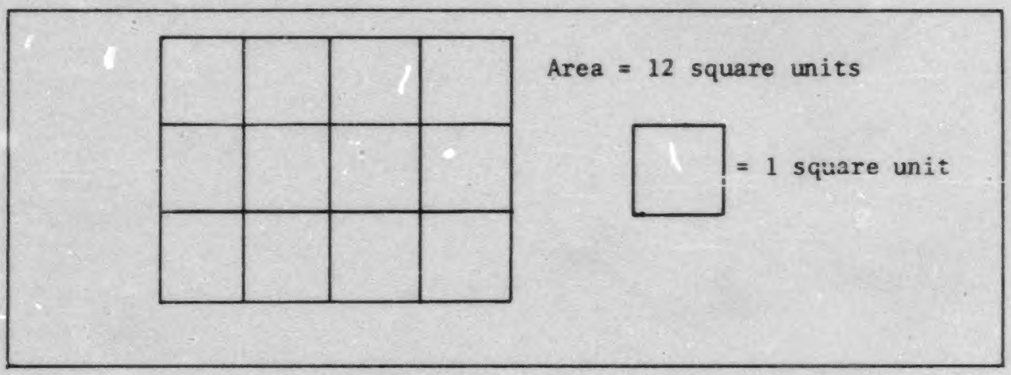
Finding Lengths, Areas, and Volumes by Counting Units

A length is given in units (inches, feet, yards, etc.) We can divide any length into units and count the number of units to see how long it is:



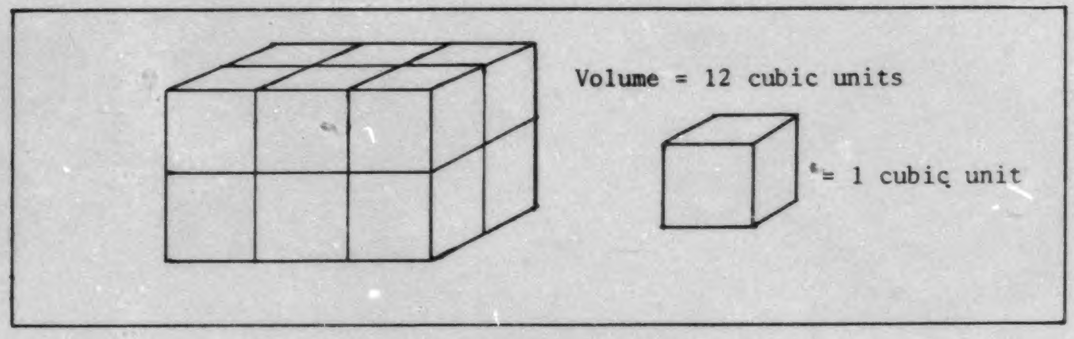
This line is ____ units long

When we measure area we count how many square units we have.


Area = 12 square units
= 1 square unit

How many square units were in the rectangle? ______

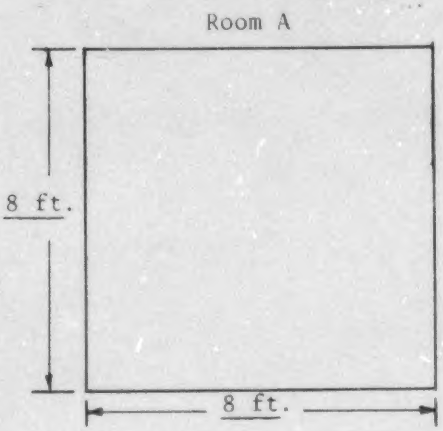
When we measure volume we count how many cubic units we have.

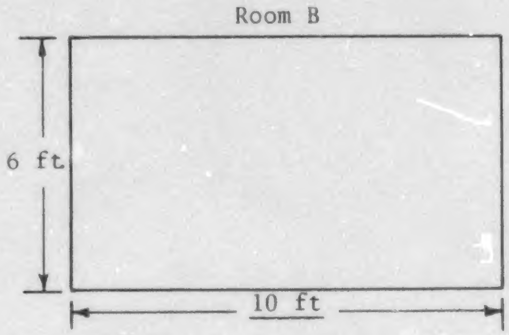

Volume = 12 cubic units
= 1 cubic unit

How many cubic units were in the stack? ______

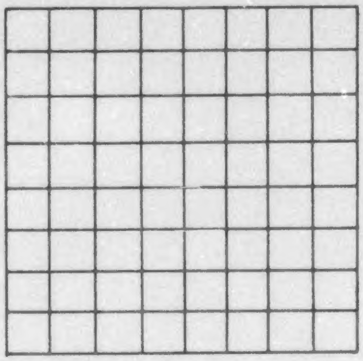
Measuring Areas--The Size of a Surface

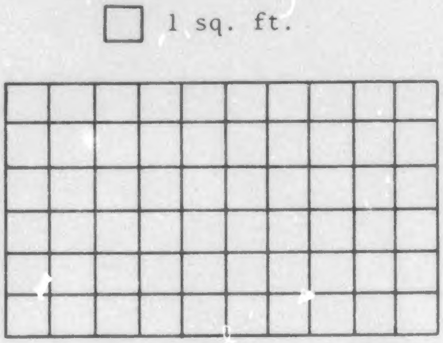
Suppose we want to compare two rooms to see which one has the larger floor space. We know how long and how wide each room is but this does not tell us which one has the larger surface or area.





If we divided each room up into little squares, we could count them and tell which room was larger.





The area of room A = ____ sq. ft.

The area of room B = ____ sq. ft.

(We call each little square one square foot because it is a square with one foot on each side. The abbreviation for square foot is: sq. ft.)

Counting by Multiplying

It is a lot of work to divide an area into square units and count them up. There is a faster way to tell how large an area is.

Suppose you have a box of eggs and want to know how many you have.

```
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
```

You could count them or--you could say "I have 5 rows with 6 eggs in each row."

5 sets of 6 is: 5 x 6 = ____

Instead of counting to find the total, it's faster to multiply the number in each row by the number of rows.

Find the number of eggs in each box by multiplying the number in each row by the number of rows. Check your answer by counting.

(H3.)

```
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
```

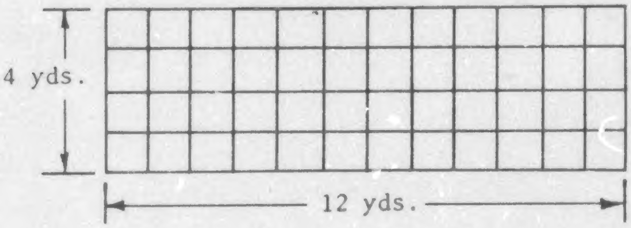
Answer = ____ eggs

(H4.)

```
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
```

Answer = ____ eggs

Just as you can count eggs by multiplying, you can count square feet or square yards or square miles by multiplying.



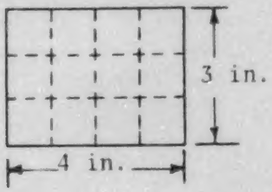
You can say "I have 4 rows with 12 in each row."

4 x 12 = ____

Each square is 1 square yard.

Since we are measuring in square yards, our answer should be written as 48 sq. yds.

We can find how many square units will fit inside a rectangle without dividing it up into parts.



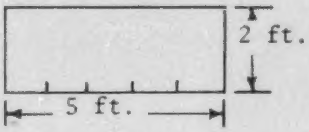
We could divide this figure into 1 in. squares to find that the area = 12 sq. in.

It is easier to say that 3 in. x 4 in. = 12 sq. in.

When we divide a rectangle into square units and multiply the number of units in each row by the number of rows, we find how many square units there are. But this is the same as multiplying the number of units in one side by the number in the other side.

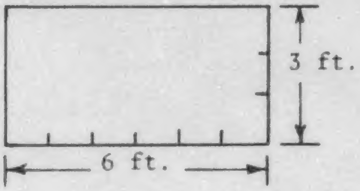
For the following problems find the area by multiplying the lengths of the sides.

Example:



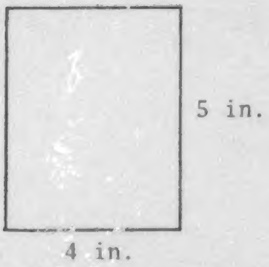
Area of a 2 ft. by 5 ft. rectangle is: 2 ft. x 5 ft. = 10 sq. ft.

(H5.)

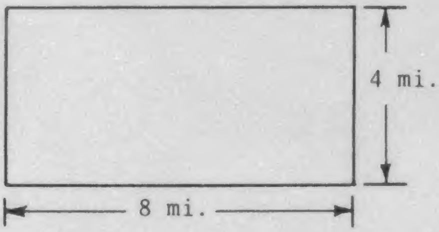


Area of a 3 ft. by 6 ft. rectangle is:
___ ft. x ___ ft. = ___ sq. ft.

(H6.)



(H7.)



Area = ___ in. x ___ in.

= ___ sq. in.

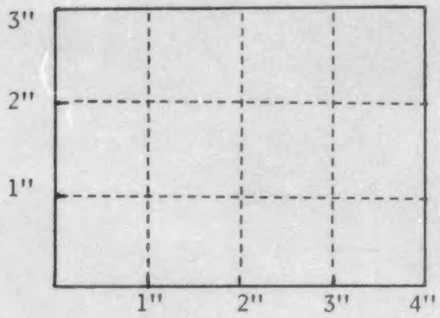
Area = ___ mi. x ___ mi.

= ___ sq. mi.

Find the area of each figure by multiplying.

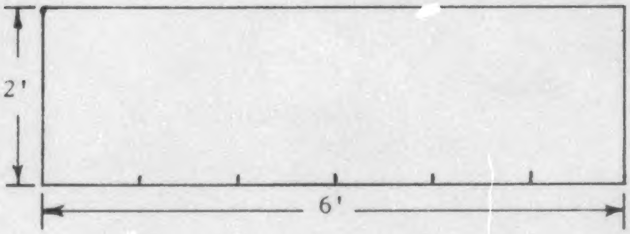
(H8.) Area = ___ sq. in.

□ = 1 square inch

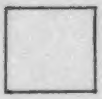


(H9.) Area = ___ sq. ft.

□ = 1 square foot



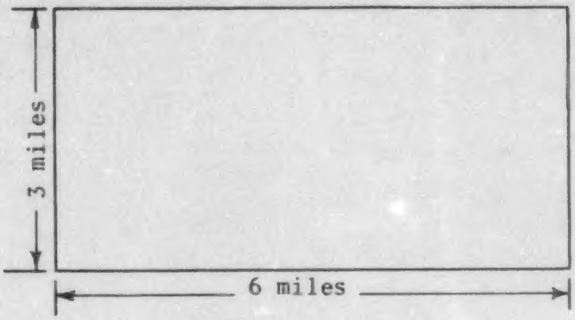
(H10.) If the below sketch is a fence around a farm, what is the area of the farm?



= 1 square mile

The area of a 3 mile by 6 mile rectangle is:

______ sq. miles



Now try these.

(H11.) A broken window is 5' x 4'. How many square feet of glass will it take to repair the window? ____ sq. ft.

Area equals ___ft.___ times ___ft.___

(H12.) The floor space in an ammunition locker aboard ship is 20 feet by 40 feet. What is the area of the floor space? Answer = ___ sq. ft.

(H13.) The construction of a house calls for 10 pieces of plywood, each piece being 8 x 12 feet. How many square feet will there be in each piece of plywood? ________ How many square feet of plywood will we need in all? ________

(H14.) It takes 500 tiles to cover a floor. Each tile is a 6" square. What is the area of each tile? ________ sq. in. What is the area of the floor? 500 x ___ = ___ sq. in.

(H15.) A square foot has 12 inches on each side. How many square inches would that be? ________ sq. in.

(H16.) A square yard has 3 feet on each side. How many square feet would that be? ________ sq. ft.

(H17.) When I find area I am counting the number of: _______ (a, b, or c)

a. sides

b. units

c. square units

Remember, to calculate an area you count the number of square units in one row and multiply by the number of rows. Area is the number of square units that a surface may be divided into.

For each problem only one answer is correct, a or b. Put an a or b in each blank to show which is right.

Example:

A tug boat is pulling 9 barges.
The deck of each barge is 50' long and 30' wide.
How many square feet of deck space does each barge have?

a. 9 x 30' x 50' = 13,500 square feet

b. 30' x 50' = 1,500 square feet Answer b

(H18.) A picture is 16" high and 20" long. How many inches of picture frame will it take to enclose the picture?

a. 16" + 16" + 20" + 20" = 72"

b. 16" x 20" = 32 square inches Answer ___

(H19.) How large is the surface of a blackboard that is 5 feet long and 3 feet high?

a. 5' + 5' + 3' + 3' = 16'

b. 5' x 3' = 15 square feet Answer ___

(H20.) The four sides of a fence around a restricted area are 40', 40', 20', and 20'. What is the area inside the fence?

a. 40' x 20' = 800 square feet

b. 40' + 40" = 20' + 20" = 120 square feet Answer ___

(H21.) Each of 4 boards is 8 x 2 feet. How many square feet is that in all?

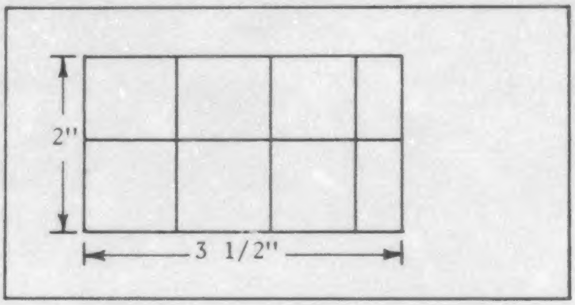
a. 8' x 2' = 16 sq. ft.

b. 4 x 16 sq. ft. = 64 sq. ft.

Answer ___

Areas with Fractions

What happens when you take the area of a rectangle with a fractional length?



The process is exactly the same. There are 3 1/2 square inches in each row and there are 2 rows.

Area = rows x number in each row

or: 2 x 3 1/2 = 7

To check on this method, count the squares. (Remember 2 halves make 1 whole.) Are there 7 squares? ____

Here is another example:

To carpet a room that is 6 yards by 4 1/2 yards, how many square yards of carpeting do you need?

Area = 6 yd. x 4 1/2 yd.

(Change the mixed number 4 1/2 to 9/2, then, multiply by 6)

6 x 9/2 = 54/2 or 27

Answer = ____ square yards

For each of the following problems find the area.

Example:

What is the area of a table top that is 3 1/2 ft. by 2 3/4 ft.?

3 1/2 = 7/2

2 3/4 = 11/4

7/2 x 11/4 = 77/8 = 9 5/8

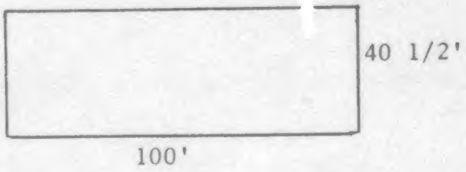
Answer = 9 5/8 sq. ft.

(H22.) What is the area of a room that is 4 1/2 yd. x 6 yd.?

____ x ____ = ____

Answer = ____ sq. yd.

(H23.) What is the area of the lot shown below?



____ x ____ = ____

Answer = ____ sq. ft.

(H24.) What is the area of a TV picture tube that is 6 1/2 by 8 inches?

____ x ____ = ____

Answer = ____ sq. in.

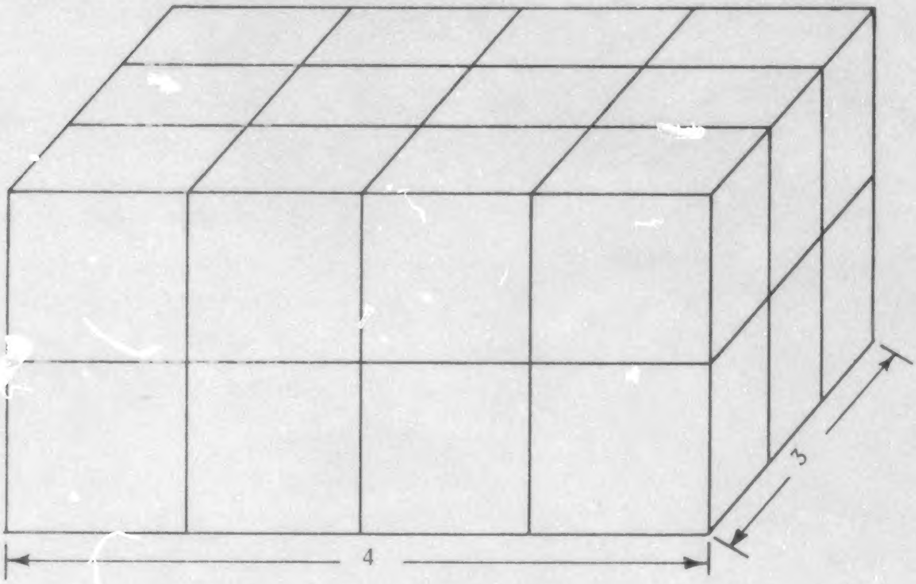
Measuring Volumes

Counting how many.

Remember that we showed we could find how many eggs are in a container by multiplying the rows by the number in each row.

How about when objects are piled on top of each other?

How many boxes are stacked up below? You can't just count them because you can't see them all. You can see, however, that there are 2 x 4 or 8 boxes in front and 2 more sets of 8 boxes in back. Three sets of 8 boxes makes 24 in all.



How could we tell there were 24 boxes without counting them?

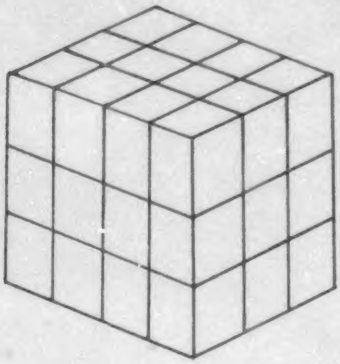
To find how many boxes are in a stack without counting them we multiply the number across the front (4) by the number stacked high (2) by the number stacked deep (3).

2 x 4 x 3 = _____ boxes

How Many in a Stack? Counting by Multiplying

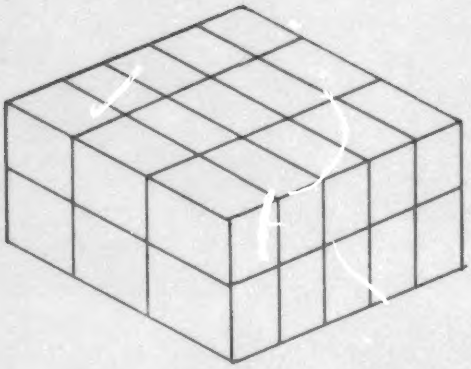
To find how many things are in a stack, we don't have to count. We can multiply.

Below are two stacks of boxes. How many are in each stack?



(H25.) The number of boxes is:

__ x __ x __ = __



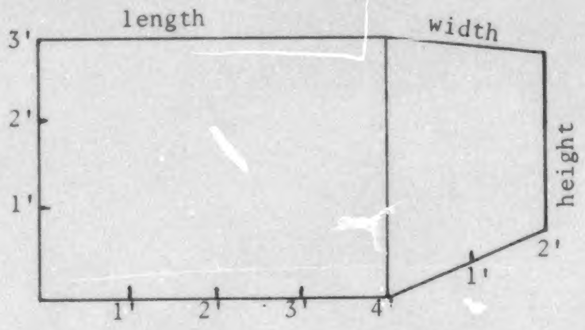
(H26.) The number of boxes is:

__ x __ x __ = __

Volume by Multiplying

Now you know how to find how many things are in a stack. You multiply the number across by the number up and down by the number deep.

How would you find the volume of the box below?

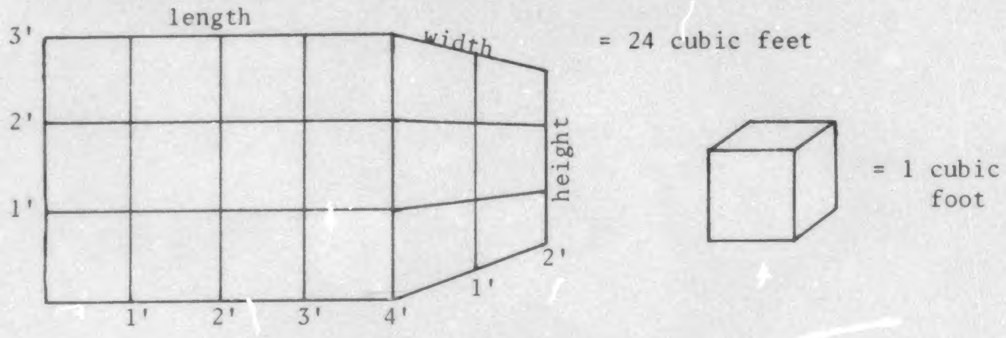

length
width
height
3'
2'
1'
1'
2'
3'
4'
1'
2'

length = 4'

height = 3'

width = 2'

You could do it by dividing the box into cubic units and counting them as we have done below.


length
width
height
3'
2'
1'
1'
2'
3'
4'
1'
2'

= 24 cubic feet

= 1 cubic foot

How many cubic units are there? ____

But we don't have to divide the box up into cubic units. We could simply multiply length x width x height and get 4 x 2 x 3 = ____ cu. ft.

The answer should be 24 either way.

Rules for Finding Volume

When you find volume you are finding how many cubic units we can stack up inside.

To find the volume, you just multiply the number in the length times the number in the height times the number in the width.

The answer to a volume problem is always in cubic units. (Cubic feet, cubic inches, cubic yards, etc.)

Try these volume problems.

(H27.) On a ship, they sometimes have to flood compartments. If a compartment was 10 feet long 12 feet high, and 14 feet wide, how many cubic feet of water would it hold?

Number of cubic feet = ____ times ____ times ____ = ____ cubic feet.

(H28.) We have two boxes. Box A is 2' wide 4' long and 3' high. Box B is 3' long, 3' wide and 3' high. Which box is larger?

Box A is ____ cubic feet.

Box B is ____ cubic feet.

Box ____ is larger.

Fractional Volumes

When you were solving for areas, you may remember, we told you that it made no difference if any of the lengths were fractional.

Length = 5 1/2"
Width = 2 1/3"
Area = 5 1/2" times 2 1/3"

This is also true for volumes. You just multiply the length times the width times the height. For example:

If a box has sides with the following dimensions:

Length = 5 1/2"
Width = 6"
Height = 4"

Then the volume is:

5 1/2" times 6" times 4" = ?

11/2 x 24 = $\frac{264}{2}$ or 132 cubic inches

Note.--The whole numbers 6 and 4 were multiplied together first to get 24 and the fraction 5 1/2 changed to 11/2.

Now work these problems.

(H29.) What is the volume of a block of wood that is 2 1/2" by 2" by 3"?

volume = ____ times ____ times ____ = ____ cubic inches.

(H30.) You car radiator is 20" long, 30" high and 3 1/2" thick. How many inches of water will your radiator hold?

____ x ____ x ____ = ____ cu. in.

(H31.) The inside of a refrigerator room is 8 yards long, 4 1/4 yards wide, and 3 1/2 yards high. How large is the space in cubic yards?

Answer = ____ cu. yd.

The answer to a measures problem may be a length, an area or a volume. The answer may be given in units, square units, or cubic units. Each of the following questions deals with how to find and give answers to measures problems.

(H32.) To find the length of a fence around a field do you add or multiply?

Answer ______________

(H33.) To find the area of a field do you add or multiply?

Answer ______________

(H34.) To find the volume of a box do you add or multiply?

Answer ______________

(H35.) If I add:

3 in. + 3 in. + 4 in. + 4 in.

do I get: 14 in., 14 sq. in., or 14 cu. in.?

Answer ______________

(H36.) If I multiply:

3 in. x 4 in.

do I get: 12 in., 12 sq. in., or 12 cu. in.?

Answer ______________

(H37.) If I multiply:

3 in. x 4 in. x 2 in.

do I get: 24 in., 24 sq. in., or 24 cu. in.?

Answer ______________

LESSON H-II

RATES AND AVERAGES

Rate of Speed

If I told you that I drove 50 miles in 2 hrs. you might say to yourself:

> "He went 50 miles in 2 hrs. so he went half as far or 25 miles in one hour. He must have been driving at 25 mph."

My rate of speed was 25 miles per hour. Rate of speed tells you how far you go in a single unit of time. (1 hr., 1 min., etc.)

Suppose I told you I went 300 miles in 6 hours. What would be my rate of speed then? Again, you want to know how far I went in 1 hour. If I went 300 miles in 6 hours, I must have gone 1/6 of 300 miles in 1 hour.

$$\boxed{1/6 \text{ of } 300 = 6\overline{)300} = 50}$$

My rate of speed must have been __________ mph.

Notice that to find my rate of speed we divided the distance that I went by the time it took to get there.

$$\boxed{\text{Rate of Speed} = \frac{\text{Distance}}{\text{Time}}}$$

You can use this formula to find your rate of speed.

Here is an example of a problem where you need to find a rate of speed.

How fast must I go to travel the 150 miles between San Diego and Los Angeles in 2 hours?

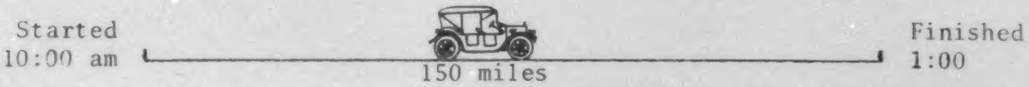
$$\text{Rate of Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{150}{2} = \underline{\hspace{3cm}}$$

I would have to go 75 mph to make it in 2 hours.

Now see if you can work these rate problems. Remember, Rate = Distance/Time.

(H38.)

Speed and Driving



Time = __________ hours Distance = __________ miles

Rate = Distance/Time = __________/__________ miles per hour

Answer = _______ miles per hour.

(H39.)

Which car is faster?

Bob's car averaged 300 miles in 5 hours.

Al's car averaged 200 miles in 4 hours.

Bob's rate = __________ mph

Al's rate = __________ mph

Find the rate for each bug by measuring the distance it goes and dividing by the time it takes. (Rate = Distance/Time) Where remainders are found write them as fractions.

Example:

4:20 pm 4:22 pm

Red Bottomed
Switch Tail

Rate = $\frac{3 \text{ inches}}{2 \text{ minutes}}$ = 1 1/2 inches/min.

(H40.)

4:20 pm ____________ 4:21 pm

Stripped Wugger

Rate = $\frac{1 \text{ inch}}{\underline{1} \text{ minute}}$ = ____ inch/min.

(H41.)

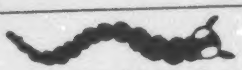
4:20 pm 4:26 pm

Green Fugwort

Rate = $\frac{4 \text{ inches}}{\_\_ \text{ minutes}}$ = ____ inches/min.

(H42.)

4:20 pm 4:23 pm



Black Wobble Bug

Rate = $\frac{3 \text{ inches}}{\_\_ \text{ minutes}}$ = ____ inch/min.

(H43.) Which of the 4 bugs had the fastest rate of speed, the Red Bottomed Switch Tail, the Stripped Wugger, the Green Fugwort, or the Black Wobble Bug?

The fastest bug was the ________________________________.

It traveled ________ inches per minute.

Just as you can talk about rate of speed, you can talk about rate of work.

For example: If you paint 200 square feet of deck in 2 hours, what is your rate of work? (How much work do you do in 1 hour?)

$$\text{Rate of work} = \frac{\text{Amount of work}}{\text{time}}$$

$$\text{Rate of work} = \frac{200 \text{ sq. ft.}}{2 \text{ hr.}} = 100 \text{ sq. ft./hr.}$$

We would say your rate of work was 100 sq. ft. per hour.

Work the following problems in the same manner:

(H44.) If Bob can send 180 words in Morse Code in 6 minutes and Dave can send 250 words in 10 minutes, who has the fastest rate?

$$\text{Bob's rate} = \frac{\text{No. words}}{\text{time}} = \frac{\_\_\_\_\text{ words}}{\_\_\_\_\text{ min.}} = \_\_\_\_\_\_\_\_\text{ words/min.}$$

$$\text{Dave's rate} = \frac{\text{No. words}}{\text{time}} = \frac{\_\_\_\_\text{ words}}{\_\_\_\_\text{ min.}} = \_\_\_\_\_\_\_\_\text{ words/min.}$$

__________ has the fastest rate.

(H45.) If a pump can remove 600 gallons of water in 3 hours, what is its rate of water removal?

Rate of work = __________ gal./hr.

The Average Amount

What is an average?

If you divide a number of things into groups so that each group has the same amount, you would have found an average. For example:

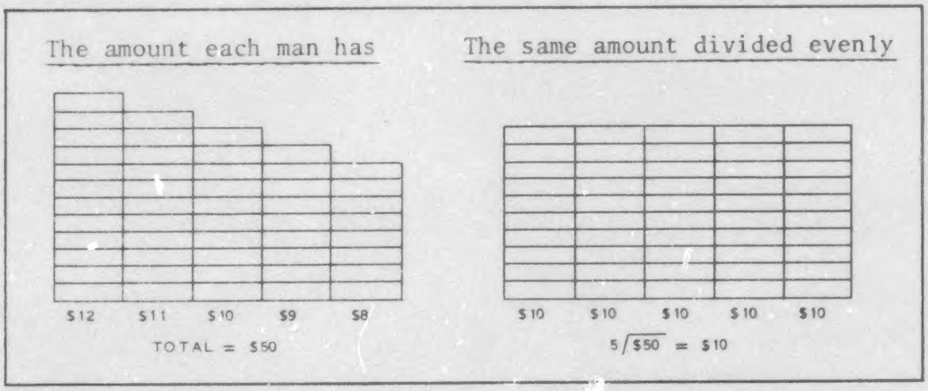
5 sailors on liberty have a total of $50 between them.

How much would each sailor have if the money was divided evenly?

$$5\overline{)50} = 10$$

Answer = $10

If you are given a number of different values, you can always find the average value by dividing the total into equal groups.


The amount each man has

The same amount divided evenly

Although each man has a different amount of money, if you

divided the $50 evenly, between them, each would have $___.

We would say that the average amount of money per man is $10.

Finding An Average

In the above problem we divided the total amount of money by the number of men to find the average amount--the amount each would have if each had the same.

To find an average, first find the total amount, then divide the total amount into even parts.

Here is an example of a problem where you would need to find a average.

Suppose you wanted to find out how much you spend on gas for your car each month. If you kept a record of what it cost each month it might look like this.

Month	Cost
January	$35
February	$20
March	$25
April	$40
May	$32
June	$28

It is clear that some months you spent much more than other months. The average cost is what you would spend if you paid the same amount each month. To find the average amount, you add up the total amount you spent, then divide by the number of months.

Total cost = ($35 + $20 + $25 + $40 + $32 + $28) = $____

Number of months = ____

Cost = number of months/ total cost

= 6 / $180 = $30

Here is another example. Remember, to find an average, first: add your values together to get a total, second: divide the total by the number of values you added together.

Find the average of the following values:

5, 6, 9, 3, 8, 7, 4

Total = 42 No. of items = 7 Average = 7/ 42 = 6

Find the average for each of the following problems.

(H46.) If you worked a total of 56 hours last week, what was your average hours of work per day?

Total = ____ hours No. of days = 7

Average = __/__ = ____ hours per day

(H47.) If you drive 300 miles Monday, 250 miles Tuesday, 400 miles Wednesday, 325 miles Thursday, and 225 miles Friday, what is your average mileage?

Total = ____ miles No. of days = ____

Average miles per day = ____

(H48.) Suppose you wanted to find out if your grade on a test was above or below the class average. The grades of the class were: 80, 64, 75, 52, 78, 60, 76, 65, 88, 62. What was the class average?

Class average = total of all grades/number of grades

Total of all grades = ____

Number of grades given = ____

Class average = ____

If your grade was 78, was that above or below the class

average? __________

(H49.) If you were playing basketball after work and in the last 6 games you played in, you shot 20 points, 32 points, 40 points, 28 points, 26 points, and 46 points, what was your average score for the 6 games?

Average score = total/number of games

Average score = ____ points

(H50.) What is the average of 7 and 21?

Total value = ____ + ____ = ____

How many numbers? ____

Average = ____

LESSON H-III

UNIT CONVERSION

What Is a Unit Conversion?

We make a unit conversion when we change hours to minutes, feet to inches, pounds to ounces, gallons to quarts, and whenever we convert one unit into another.

How To Make Unit Conversions

Before you can change one unit to another, you have to know the relationship between the units. Some you know because you use them all the time.

(1 year = ____ months, 1 foot = ____ inches, 1 day = ____ hours)

Some units are less familiar and we have to use tables to remind ourselves what they are equal to. Use the Table of Measures on the next two pages to do the following unit conversions:

(1 gross = ____ dozen, 1 rod = ____ feet, 1 leap year = ____ days).

Changing Units By Multiplication Or Division

When you change from one unit to another you either multiply or divide.

To change to a smaller unit--multiply	To change to a larger unit--divide
4 ft. = ? in.	15 ft. = ? yd.
4 x 12 = 48	15/3 = 5

These are the only two things that you do when you change the size of your units.

In the following section you will learn how to decide whether you should multiply or divide in order to change your unit.

Table of Measures

Distance Measure

12 inches (in. or ") = 1 foot (ft. or ')
3 feet or 36 inches = 1 yard (yd.)
5½ yards or 16½ feet = 1 rod (rd.)
220 yards or 1/8 mile = 1 furlong (fur.)
320 rods or 8 furlongs = 1 mile (mi.)
1760 yards = 1 mile
5280 feet = 1 mile

Liquid Measure

2 measuring cups = 1 pint (pt.)
16 fluid ounces (fl. oz.) = 1 pint (pt.)
4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
32 fluid ounches = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

Time Measure

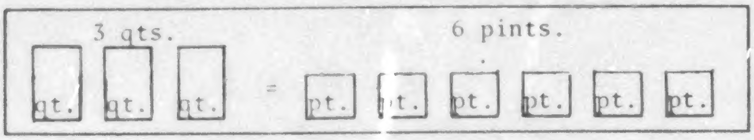
60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 month (mo.) (for interest calculations, et
360 days = 1 year (yr.) ("   "   "   "
12 months = 1 year (yr.)
365 days = 1 year (yr.)
366 days = 1 leap year
10 years = 1 decade
100 years = 1 century (C)

Units of Counting

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gr.)
144 units = 1 gross (gr.)
24 sheets = 1 quire
480 sheets = 1 ordinary ream
500 sheets = 1 printer's ream

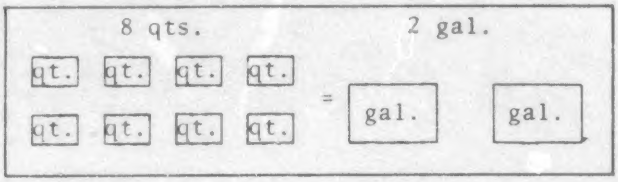
How do you decide whether to multiply or divide when changing units?

To have the same amount in smaller units you must have more units so you multiply. For example:



To change 3 quarts into pints we ________ 3 by 2 and get 6.

To have the same amount in larger units you must have less units so you divide. For example:



To change 8 quarts into gallons we ______________ 8 by 4 and get 2.

To summarize, if you are changing to larger units you will have less -- divide. If you are changing to smaller units, you will need more -- multiply.

How do we decide what number to multiply or divide by?

Your first step is to decide what the relation is between your units. For example: Change 180 minutes into hours.

1 hr. = 60 min.
so:
180 min. = 180/60 hrs.
= 3 hrs

To change smaller units to larger units we had to divide.

We divided by ____ because 1 hr. = 60 min

Complete each unit conversion:

Example:

Change 96 hours into days.

Since we are changing to larger units we will divide.

1 day = 24 hr.   96/24 = 4 days

(H51.) Change 25 yards into feet.

Since we are changing to smaller units we will _______.

1 yd. = 3 ft.

25 yd. = 25 x 3 ft. = ____ ft.

(H52.) Change 44 quarts into gallons.

Since we are changing to larger units we will ________.

1 gal. = 4 qt.

44 qts. = 44/4 gal. = ____ gal.

(H53.) Change 2 1/2 hours into minutes.

Since we are changing to __________ units we will multiply.
1 hr. = 60 min

2 1/2 hr. = 2 1/2 x ____ min. = ____ min.

(H54.) Change 21 days into weeks. (Do we multiply or divide?)

1 week = 7 days   Answer: 21 days = ____ weeks

(H55.) Change 12 feet into yards. (Do we multiply or divide?)

1 yard = 3 feet   Answer: 12 ft. = ____ yd.

(H56.) Change 5 feet into inches. (Do we multiply or divide?)

1 foot = 12 inches   Answer: 5 ft. = ____ in.

Changing Parts of Measures

Sometimes we can simplify a measurement by changing parts of it into larger units.

For example, suppose we add two lengths and get an answer of 2 ft. 52 in. We can change some of the inches into feet.

2 ft. 52 in.
equals: 2 ft. + (4 ft. 4 in.)
or: 6 ft. 4 in.

To work this problem we changed the 52 in. to 4 ft. ____ in.

In the following problems we are changing parts of measures into larger units. Complete the changes. (If you can't remember how many of one unit are equal to another, check back with the Table of Measures.)

(H57.) 2 hrs. 65 min.
equals: 2 hrs. + (1 hr. 5 min.)
or: _ hrs. 5 min.

(H58.) 3 lbs. 33 oz.
equals: 3 lbs. + (2 lbs. 1 oz.)
or: _ lbs. _ oz.

(H59.) 5 ft. 25 in.
equals: 5 ft. + (2 ft. _ in.)
or: _ft. _ in.

(H60.) 2 weeks 30 days
equals: 2 weeks + (___ weeks ___ days)
or. _ weeks _ days

For each of the following problems indicate if the two values are equal by marking yes or no.

Example: 2 yd. 7 ft. = 4 yd. 1 ft. Yes

(H61.) 1 ft. 7 in. = 3 ft. 1 in. ___

(H62.) 2 hrs. 76 min. = 3 hrs. 16 min. ___

(H63.) 7 min. 125 sec. = 8 min. 5 sec. ___

Borrowing Units

Another situation where you have to change parts of one unit into a different unit is when you have to borrow to subtract.

	3 ft. 2 in.	change	2 ft. 14 in.
-	1 ft. 6 in.	to	- 1 ft. 6 in.
			1 ft. 8 in.

We had to change 3 ft. 2 in. to __ ft. __ in.

Complete the following changes.

(H64.) 5 ft. 4 in. = 4 ft. __ in.

(H65.) 2 hrs. 15 min. = 1 hr. __ min.

(H66.) 3 days 11 hrs. = 2 days __ hrs.

Work each of the following problems, changing units where necessary.

(H67.)	6 hrs. 15 min.	change		5 hrs. __ min.
	- 2 hrs. 30 min.	to	-	2 hrs. 30 min.
				3 hrs. __ min.

(H68.)	5 weeks 3 days	change		__ weeks __ days
	- 1 week 6 days	to	-	1 week 6 days
				__ weeks __ days

(H69.)	5 yd. 1 ft.	change		__ yd. __ ft.
	- 2 ft.	to	-	2 ft.
				__ yd. __ ft.

(H70.)	10 hrs. 5 min.	change		__ hrs. __ min.
	- 4 hrs. 20 min.	to	-	4 hrs. 20 min.
				__ hrs. __ min.

Watching for Unit Switches in Problems

The people who make tests can be pretty sneaky. Very often, they will toss in a problem that calls for a switch of units. They can foul you up if you aren't watching out.

One way a test maker can trick you is by switching from one unit to another. For example:

> If a man is running at a rate of 2 feet per second for 60 seconds, how many yards does he run?

A student working this problem might say: 2 times 60 = 120

Answer = 120 yards

Would this answer be correct? ________

The student fell into a trap. He didn't notice that the problem was given in feet per second, but the answer was asked for in yards per second. He should have said:

> "2 x 60 = 120. The runner goes 120 feet. But 120 feet = 40 yards."

Answer = ____ yards

A test maker can trick you into switching between units and decimals. For example:

A man drives from 8:45 am to 11:15 am one morning. How long does he drive?

A student might say:

"We need to know how many hours he was driving. This would be:"

$$\begin{array}{lrl} & & 11{:}15 \text{ hours} \\ & - & \underline{8{:}45 \text{ hours}} \\ \text{equals} & & 2{:}70 \text{ hours or 3 hrs. 10 min.} \end{array}$$

The student has goofed again. He made his error when he tried to work the problem as if he had decimal numbers.

What does 11:15 mean? ____ hours and ____ minutes.

What does 8:45 mean? ____ hours and ____ minutes.

Before we can subtract we have to borrow and convert units.

The problem should be worked like this:

$$\begin{array}{rl} & 11 \text{ hrs. } 15 \text{ min.} \\ - & \underline{8 \text{ hrs. } 45 \text{ min.}} \end{array} \quad = \quad \begin{array}{rl} & 10 \text{ hrs. } 75 \text{ min.} \\ - & \underline{8 \text{ hrs. } 45 \text{ min.}} \\ & 2 \text{ hrs. } 30 \text{ min.} \end{array}$$

In order to subtract we changed 11 hours 15 minutes to

10 hrs. ____ min.

Work each of the following problems changing units where necessary.

(H71.) From 9:30 am to 11:20 am is ? hrs. ? min.

11 hrs. 20 min.
- 9 hrs. 30 min.

= ___ hrs. ___ min.
- 9 hrs. 30 min.
___ hrs. ___ min.

(H72.) From 8:16 am to 10:45 am is ___ hrs. and ___ min.

(H73.) Subtract 8 ft. 10 in. from 12 ft. 3 in.

___ ft. ___ in.
- ___ ft. ___ in.

Answer = ____ ft. ____ in.

(H74.) Which answer is correct? ____

3 ft. 11 in.
+1 ft. 8 in.

=
(a) 3 ft. 19 in.
(b) 5 ft. 9 in.
(c) 5 ft. 7 in.
(d) None of these

(H75.) Which answer is correct? ____

2 hr. 30 min.
+ 1 hr. 55 min.

=
(a) 35 min.
(b) 3.85 hrs.
(c) 4 hrs. 25 min.
(d) None of these

(H76.) If a man travels 6 ft. per second, how many yards will he go in 20 min.?

If in one second he goes 6 ft.

Then in one minute he goes 60 x 6 ft. = ____ ft.

or: ____ yd.

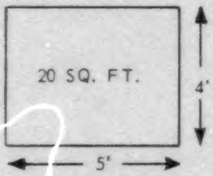
In 20 minutes he goes 20 x _____ yd. = ____ yd.

LESSON H-IV

MEASUREMENT FORMULAS

Splitting Rectangles into Triangles

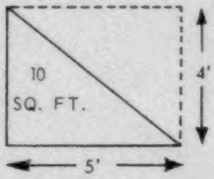
You remember how to find the area of a rectangle. You multiply the length by the width to find the number of square units that will fit.



Area = length x width

Area = 4' x 5' = 20 sq. ft.

If you divide a rectangle into two square cornered triangles, the area of each triangle will be half of the area of the rectangle.



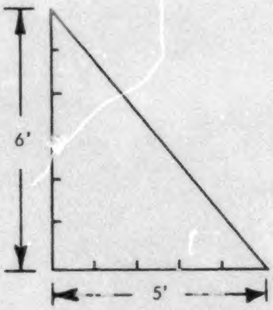
Area of rectangle = 20 sq. ft.

Area of traingle = 10 sq. ft.

The area of the rectangle was ____ sq. ft.
The area of the triangle was half as much or ____ sq. ft.

To find the area of a triangle with a square corner, make the triangle into a rectangle. The area of the rectangle will be twice as much as the area of the triangle.

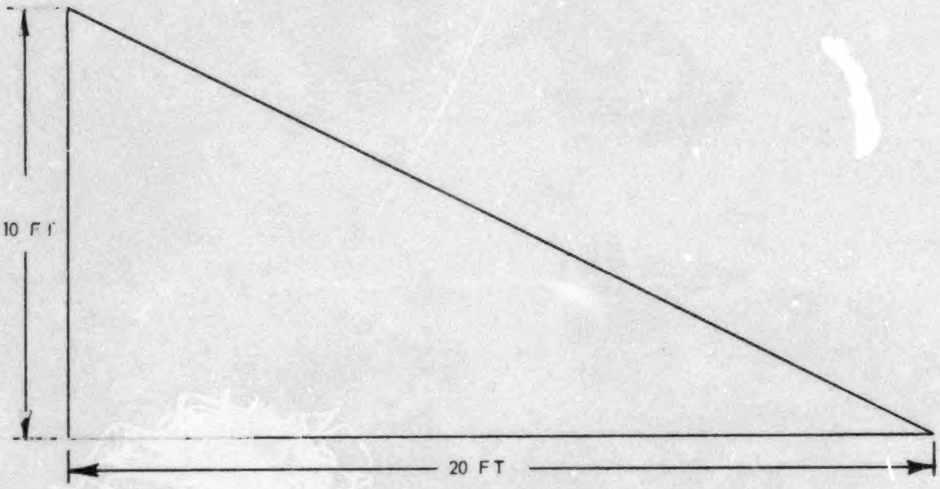
(H77.)



If this triangle was made into a rectangle, what would the area of the rectangle be? ____ sq. ft.

What would be the area of the triangle? ____ sq. ft.

(H78.)



The area of this triangle = ____ sq. ft.

Formulas for Areas and Volumes

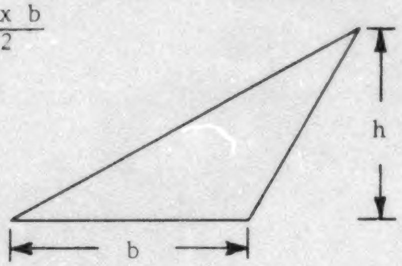
For some shapes, there is no easy way to remember how to find an area or volume. In this case, you would have to get a math book and look up the formula.

Formulas for Areas of Triangles

You can find the area of a triangle with square corners by changing it into a rectangle. What can you do if the triangle doesn't have any square corners?

You can use this formula to find areas of triangles.

$$A = \frac{h \times b}{2}$$



h = height of the triangle

b = length of the bottom or base

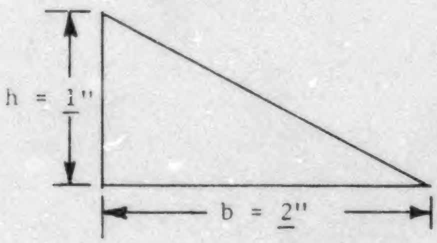
This triangle does not have a square corner and, therefore, can't be changed into a rectangle. Even so, we can use our formula.

If the base (b) = 4" and the height (h) = 3" what would be the area?

$$A = \frac{h \times b}{2} = \frac{4 \times 3}{2} = \frac{12}{2} = \_\_\_\_ \text{ sq. inches}$$

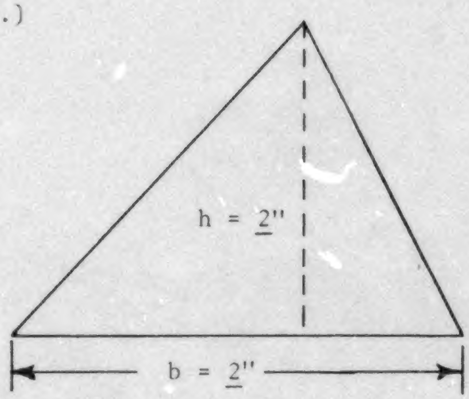
Using the formula $A = \frac{hb}{2}$ find the area of these triangles.

(H79.)



$A = \frac{hb}{2} = \frac{(\ )\ x\ (\ )}{2} =$ ____ sq. in.

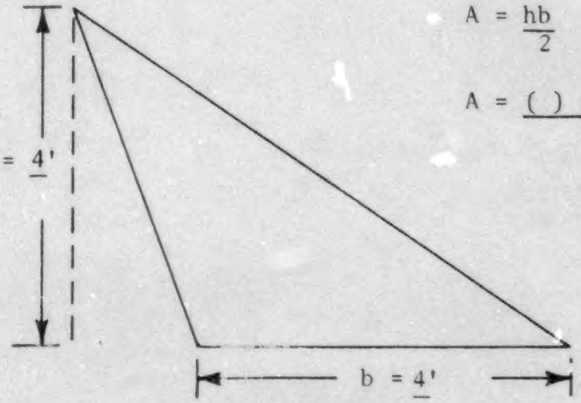
(H80.)



$A = \frac{hb}{2}$

$A = \frac{(\ )\ x\ (\ )}{2} =$ ____ sq. in.

(H81.)

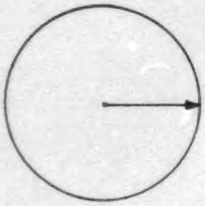


$A = \frac{hb}{2}$

$A = \frac{(\ )\ x\ (\ )}{2} =$ ____ sq. ft.

Formulas for Areas of Circles

The radius (r) of a circle is the distance from the center to the edge.



If you know the length of the radius you can find the area with the formula:

$A = \pi r^2$

(π is always equal to 3.14)

In each of the following problems you are told how long the radius of a circle is. Use the formula $A = \pi r^2$ to find the surface area of each circle. (Remember r^2 means r x r.)

Example:

If radius = 5" r^2 = 25 sq. in.

Then: πr^2 is: 3.14 x 25

Area = 78.5 sq. in.

(H82.) If radius = 3 in. r^2 = 9 sq. in.

Then: πr^2 is: 3.14 x ____

Area = ____ sq. in.

(H83.) If radius = 10' r^2 = ___ sq. ft.

Then: πr^2 is: ____ x ____

Area = ____ sq. ft.

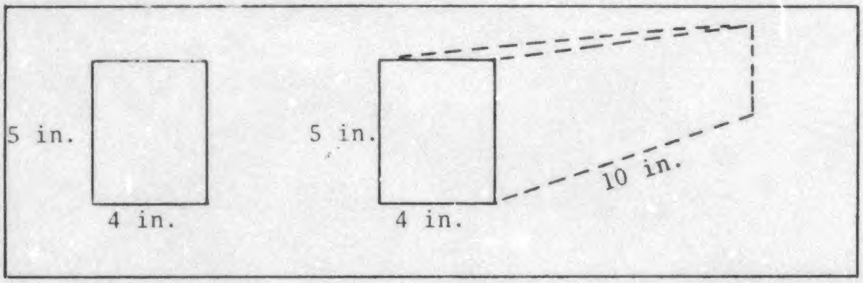
(H84.) If radius = 2 yd. r^2 = ___ sq. yd.

Then: πr^2 is: ____ x ____

Area = ____ ______

Finding Volumes from Areas

To find the area of a rectangle, you multiply the lengths of the two sides together. If you extend the area to get a solid figure you have a third length.



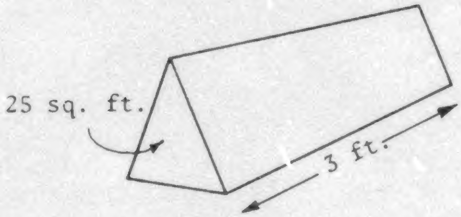
Multiplying the 3 lengths gives you the volume.

Multiplying the area by the 3rd length also gives you the volume.

Volume = (4 in. x 5 in.) x 10 in. = 200 cu. in.

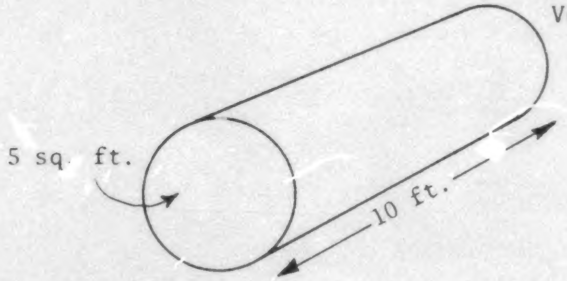
Volume = (20 sq. in.) x 10 in = 200 cu. in.

Whenever an area is extended to make a solid figure, you can multiply the area times the length to get the volume. This is true no matter what the shape of the area. For example:



Volume = 2 x 3 cu. ft.

= _____ cu. ft.



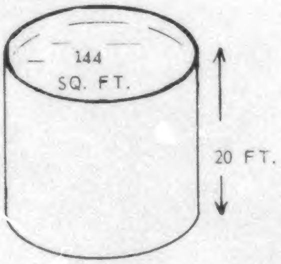
Volume = __ x __ cu. ft.

= 50 cu. ft.

Work the following problems using the area to find the volume.

Practice Problem:

Suppose you wanted to know how many cubic feet of water a water tank will hold. You measure the area of the top and find it is 144 square feet. The height of the tank is 20 feet.



Volume = area x height.
Volume = 144 sq. ft. x 20 ft.
or: ____ cu. ft.

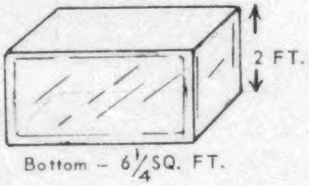
We can say the tank will hold 2880 cu. ft. of water.

(H85.) How many cubic inches of water will fit in 100 feet of pipe

if the area of the opening = 5 sq. ft.?

Volume = ____ cubic ft.

(H86.) How many cubic feet of water will this fish tank hold if the surface area of the bottom is 6 1/4 sq. ft. and the tank is 2 ft. deep?



Volume = ____ cu. ft.

LESSON H-V

CONVERSION OF SQUARE UNITS AND CUBIC UNITS

When you want to change feet into inches you multiply by 12.

For example, 5 ft. = ? in.

1 ft. = 12 in.

so: 5 ft. = 5 x 12 in. = 60 in.

To work this problem, you had to know that 1 ft. = 12 in.

When you want to change feet into yards you divide by 3.

3 ft. = 1 yd.

so: 6 ft. = $\frac{6 \text{ yd.}}{3}$ = 2 yd.

To work this problem you had to know that 3 ft. = ____ yd.

What about changing square units or cubic units?

Again, you have to know how many of the smaller units are in one of the larger units. These are different than for plain units. For example:

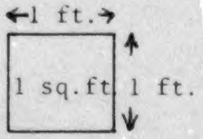
1 ft. = 12 in. but 1 sq. ft. does not equal 12 sq. in.

3 ft. = 1 yd. but 3 cu. ft. does not equal 1 cu. yd.

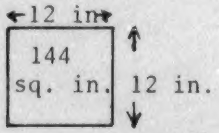
Changing Square Units

When you change larger square units to smaller square units you have two lengths to change.

This is a square foot.



This is a square foot changed into square inches.



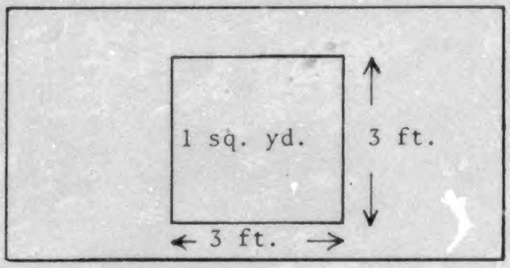
You remember that in order to find the area of a square you multiply the length by the width. A square foot is a square whose sides are each 1 ft. long. Since 1 ft. = 12 in. each side of a square foot is 12 in. long. The area of a square foot is:

12 in. x 12 in. = ____ sq. in.

so: 1 sq. ft. = 144 sq. in.

Work the following problem.

(H87.) There are 3 feet in one yard. A square yard would be a square whose sides are each ____ feet long.

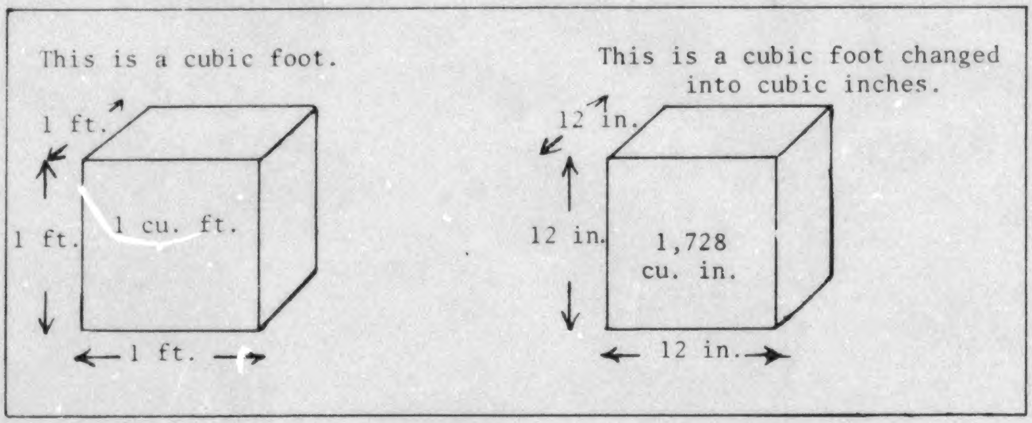


The area of a square yard would equal __ ft. x __ ft.

or: __ sq. ft.

Changing Cubic Units

When you change larger cubic units to smaller cubic units you have 3 lengths to change.


This is a cubic foot.

This is a cubic foot changed into cubic inches.

Each side of a cubic foot is ____ in. long

Volume = length x width x height

The volume of a cubic foot = 12 in. x 12 in. x 12 in.

so: 1 cu. ft. = ____ cu in.

Work the following problem changing large cubic units into small cubic units.

How many cubic inches are in 10 cubic feet?

1 cubic foot = __ x __ x __ = ____ cu. inches

10 cubic feet = 10 x ______ cubic inches

10 cubic feet = ____ cubic inches

To change any number of square or cubic units to other units, first decide how many smaller units are in one larger unit. For example:

5 sq. yds. = _?_ sq. ft.

1 sq. yd. = 3 ft. x 3 ft. = 9 sq. ft.

5 sq. yds. = 5 times 9 sq. ft. = 45 sq. ft.

We know that a square yard is a square with each side 3 ft. long. The area of a square yard must be 3 x 3 or: 9 sq. ft.

Since 1 sq. yd. = 9 sq. ft., then 5 sq. yd. = ____ sq. ft.

Complete the following unit changes.

(H88.) 2 sq. ft. = _?_ sq. in.

1 sq. ft. = 12 in. x 12 in. = ____ sq. in.

2 sq. ft. = ___ x 144 sq. in. = ____ sq. in.

(H89.) 54 cu. ft. = _?_ cu. yds.

1 cu. yd. = 3 ft. x 3 ft. x 3 ft. = __ cu. ft.

Since 27 cu. ft. = 1 cu. yd.

54 cu. ft. = __ cu. yds.

(H90.) 10 sq. yds. = _?_ sq. ft.

1 sq. yd. = ___ sq. ft.

10 sq. yds. = ___ sq. ft.

LESSON J-I

SOLVING RATIOS

What is a Ratio?

If I say that the Navy has 10 sailors for each officer, then I have given a ratio.

1 officer to 10 sailors

Suppose this ratio is the same for each ship. If you knew how many officers were on a ship you could find out how many sailors were on it.

If	Then
10 officers	100 sailors
15 officers	150 sailors
20 officers	200 sailors

You can see that in each case we have 10 times as many sailors as officers.

If we had 50 officers, we would have 10 x 50 = ____ sailors.

We can state any ratio in two ways. In the above example we could have said.

The ratio of sailors to officers

is: 10 to 1

or we could have said:

The ratio of officers to sailors

is: 1 to 10

When you have a ratio, then knowing the value of one thing tells you the value of the other.

A certain school has 2 boys for each girl. We can say:

The ratio of: boys to girls
is: 2 to 1

Suppose this ratio was the same for each class.

	If a class has:	There will be:
	10 boys	5 girls
(J1.)	20 boys	__ girls
(J2.)	40 boys	__ girls
(J3.)	20 girls	__ boys
(J4.)	50 girls	__ boys

In each case there must be twice as many boys as girls.

Notice that whichever value we are given, the number of girls or the number of boys, we can find the other value by using our ratio.

Solving Ratios as Equations

Suppose that Sam always spends twice as much as Bill when they go on liberty together. We could write this as a ratio:

Bill's spending		1
to	=	to
Sam's spending		2

If we know what one spends we can find out what the other spends.

Suppose Sam spent $50 last night. Then:

Bill's spending		1		what ?
to	=	to	=	to
Sam's spending		2		$50

We can now find what Bill spent by changing our ratios into an equation.

$$\boxed{1/2 = n/50}$$

What is the missing value?

If: 1/2 = n/50 Then: n = 1/2 x 50 = ____

Our missing value was 25. So Bill spent $____.

Here is another example of how to solve a ratio problem by setting it up as an equation.

For every $10 Bob earns, he saves $1.

Saves 1
to = to
earns 10

How much does he save if he earns $40?

$\frac{1}{10} = \frac{n}{40} \qquad n = 40 \times \frac{1}{10} = 4$

If he earns $40 he saves $4

(J5.) How much does he save if he earns $20?

$\frac{1}{10} = \frac{n}{20} \qquad n = 20 \times \frac{1}{10} = \underline{\qquad}$

If he earns $20 he saves $____

(J6.) How much does he save if he earns $100?

$\frac{1}{10} = \frac{n}{100} \qquad n = \underline{\quad} \times \frac{1}{10} = \underline{\qquad}$

If he earns $100 he saves $____

(J7.) How much does he save if he earns $50?

$\frac{1}{10} = \frac{n}{50} \qquad n = \underline{\quad} \times \frac{1}{10} = \underline{\qquad}$

If he earns $50 he saves $____

Complete each of the following ratio problems by solving equations to find the missing value.

Example:

A salesman gets to keep $2.70 for each $9 worth of goods he sells. If he sells a $25 watch, how much does he keep?

2.70		what
to	=	to
9		25

2.70/9 = n/25 so: n = 25 x 2.70/9 = 7.50

Answer = $7.50

(J8.) If you drive 600 miles in 2 days, how far will you drive in 3 days?

600		what
to	=	to
2		3

600/2 = n/3 so: n= _____ x 600/2 = _____

Answer = _____miles

(J9.) A recipe calls for 2.5 gallons of tomatoes to feed 120 men. If a ship has a crew of 480 men, how many gallons are needed?

2.5		what
to	=	to
120		480

2.5/120 = n/480 so: n = ____ x ____ = ____

Answer = ____ gallons

(J10.) The Navy gives 30 days of leave for each 12 months spent in the service. If you have served 8 months, how much leave have you earned?

30		what
to	=	to
12		8

30/12 = n/8 so: n = ___ x ___ = ____

Answer = __ days leave

Complete the following ratio problems by changing the ratios to equations.

(J11.) If you drive 100 miles in 1.5 hr. and continue at the same speed, you will cover ____ miles in 3 hrs.

100		what
to	=	to
1.5		3

(J12.) If you smoke 3 packs of cigarettes in 2 days, then you must smoke ____ packs each day.

3		what
to	=	to
2		1

(J13.) If 3 ft. = 1 yd., then: 7 yds. = ____ ft.

3		what
to	=	to
1		7

Ratio Equations with Fractions

Sometimes there will be fractions in the ratios. This means you will have to multiply and divide by fractions to find the missing value. For example:

Given a ratio with fractions, we change it into an equation and simplify to find the missing value.

10		what
to	=	to
1 1/2		3/4

When we set up our equation, we get: 10 over 1 1/2, which means: 10 divided by 1 1/2.

Since 10 divided by 1 1/2 = 20/3, we can simplify our equation.

If: $\frac{10}{1\ 1/2} = \frac{n}{3/4}$

Then: $n = 3/4 \times \frac{10}{1\ 1/2}$

So: $n = 3/4 \times 20/3$

Now we can simply multiply and find the missing value.

$3/4 \times 20/3 = 60/12$ or 5

Putting the missing value back into our ratio, we Have:

10		5
to	=	to
1 1/2		3/4

For each of the following ratio problems, find the missing value by setting up an equation. Simplify by dividing wherever you have fractions in your ratio.

(J14.)

5		?
to	=	to
1/2		4

If: $\frac{5}{1/2} = \frac{n}{4}$

$\frac{5}{1/2} \times 4 = n$

$10 \times 4 = n$

____ = n

The missing value is: ________.

(Note that $\frac{5}{1/2}$ is 5 ÷ 1/2 which equals 10.)

(J15.)

3 ?

to = to

3/4 1/4

If: $\frac{3}{3/4} = \frac{n}{1/4}$

$\frac{3}{3/4} \times 1/4 = n$

____ x 1/4 = n

The missing value is: ________.

____ = n

(Note that $\frac{3}{3/4}$ is 3 ÷ 3/4 which equals _____.)

(J16.)

3/8 ?

to = to

1/8 2

If: $\frac{3/8}{1/8} = \frac{n}{2}$

$\frac{3/8}{1/8} \times 2 = n$

____ x 2 = n

The missing value is: ________.

____ = n

(Note that $\frac{3/8}{1/8}$ is 3/8 ÷ 1/8 which equals _____.)

The following word problems involve ratio equations with fractions. Complete the problems to find the missing values.

(J17.) On a map, two towns are 1 1/4 inches apart. If the map is scaled so that 4 inches = 800 miles, how many miles apart are the towns?

800 ?

to = to

4 1 1/4

If: $\frac{800}{4} = \frac{n}{1\ 1/4}$

Then: $n = 1\ 1/4 \times \frac{800}{4}$

But: (800/4 = ____)

And: (1 1/4 = __/4)

So: n = 5/4 x 200 or ____

Answer = ____ miles

(J18.) A ship's recipe for stew says to use 1/4 lb. of salt if you make enough stew to feed 100 men. This means you would have to use 2 3/4 lb. of salt if you are going to make enough to feed ___ men.

100		?
to	=	to
1/4		2 3/4

If: $\frac{100}{1/4} = \frac{n}{2\ 3/4}$

Then: $n = \_\_\_\_ \times \frac{100}{1/4}$

($\frac{100}{1/4}$ = ____)

So: n = ____ x ____ = ____

(2 3/4 = __/4)

Answer = ____ men

(J19.) A Photographer's Mate has set up an enlarger so that a 3/4 inch photo will be enlarged to 2 inches. At this setting, how large will a photo have to be so that it will be 2 2/3 inches after it is enlarged?

3/4		?
to	=	to
2		2 2/3

If: $\frac{\_\_\_\_}{\_\_\_} = \frac{n}{\_\_\_}$

Then: n = ___ x ___

(Simplify mixed numbers and fraction ratios.)

So: n = ___ x ___ = ___

Answer = ____ inches

LESSON J-II

SETTING UP RATIO EQUATIONS

Identifying the Ratio Equation

Often a ratio problem will be given as a word problem and you will have to translate it to a ratio equation.

To set up a ratio equation:

1. Identify the two factors you are working with.
2. Set up a ratio with the two values that are given.
3. Set up a ratio with the missing value you are trying to find.

For example:

If you walk 10 miles in 2 hours how far will you walk in 7 hours?

1. Factors		2. Given values ratio		3. Missing value ratio
miles		10		what
to	=	to	=	to
hours		2		7

Once you have set up your ratios, you can change them into an equation and find the missing value.

Equation	Answer
$\frac{10}{2} = \frac{n}{7}$	$n = \frac{10}{2} \times 7 = 35$

The answer is that you can walk ___ miles in 7 hours.

Finding the Missing Value

A ratio equation has four values. For example:

2 6
to = to
3 9

In a ratio problem, three of the four values will be given. You will have to find the fourth.

2 what
to = to
3 9

Before you can find the missing part in a ratio problem you have to be able to set up the values in a ratio equation.

Once you have set up the ratio problem, it is easy to change it into an equation.

Ratio Problem	Equation
10 what to = to 2 7	$\frac{10}{2} = \frac{n}{7}$

The trick then is in learning how to set up the ratio problem.

In order to set up the values in a ratio equation, you have to decide what factors are involved. For example:

> For every dollar that you spend there is a 6 cent sales tax.
> If you spend $1.50 how much is the tax?

Your factors are (1) what was spent, and (2) the tax.

First you would set up the factors.

tax
to
amount spent

Next you would make a ratio with the two related values that were given. (6¢ tax for $1 spent.)

tax .06
to = to
amount spent 1.00

Then we make another ratio with the other given value and the missing value.

tax .06 what
to = to = to
amount spent 1.00 1.50

Set up these ratio equation problems. (Do not try to find the missing value.)

Example

If for every $20 that you spend you must pay $3 in sales tax, how much would you spend to pay $5 in tax?

Spend		20		what
to	=	to	=	to
tax		3		5

(J20.) An apple picker finds that 2 out of every 7 apples he picks are rotten. If he picks 375 apples how many are rotten?

Rotten apples		____		what
to	=	to	=	to
apples picked		7		____

(J21.) Two out of 5 men smoke cigarettes. If there are 100 men in a movie house how many will be smokers?

smokers		____		what
to	=	to	=	to
all men		____		100

(J22.) If a working party can load 20 crates in 7 minutes, how long will it take them to load 300 crates? (Set up a ratio of minutes to crates)

____		____		what
to	=	to	=	to
____		____		____

Placing the Values

When you set up your ratio equations, you will find them easier to work with if you place the factor with the missing value at the top of the equation. For example:

> For every \$10 Bill earns he saves \$2. How much does he save if he earns \$560?

We could set up the problem like this:

earns		10		560		
to	=	to	=	to	or:	$\frac{10}{2} = \frac{560}{n}$
saves		2		what		

The only problem is that the unknown value (n) is on the bottom of the equation. While this problem could still be worked this way, it is simpler to set up the factors so the missing value is on top.

saves		___		what		
to	=	to	=	to	or:	$\frac{2}{10} = \frac{n}{560}$
earns		___		___		

Now it is easier to find the missing value.

$$\boxed{\begin{aligned} n &= 560 \times \frac{2}{10} \\ n &= 112 \end{aligned}}$$

Types of Ratio Problems

There are many types of problems that can be worked with ratios. The examples on the following pages will help you to recognize some of the types of ratio problems you may run into.

Scale Drawing Problems

Below is an example of a scale drawing problem that has been set up as a ratio.

On a map .25 inches is equal to 5 miles. If two cities were 2 inches apart on the map, what would be the real distance between them?

$$\begin{array}{ccccc} \text{miles} & & 5 & & \text{what} \\ \text{to} & = & \text{to} & = & \text{to} \\ \text{inches} & & .25 & & 2 \end{array}$$

Note that we put the missing value on top. This gives an equation that is easier to work.

Next, we change the ratio into an equation and find the missing value.

$$\frac{5}{.25} = \frac{n}{2} \qquad n = 2 \times \frac{5}{.25} = \underline{40}$$

Answer = 40 miles

Set up each problem as a ratio, then change the ratio to an equation and find the missing value.

(J23.) If a map is drawn to a scale of 1 inch = 6 miles, then 3 inches would equal how many miles?

$$\begin{array}{ccccc} \text{miles} & & \_\_\_\_ & & \text{what} \\ \text{to} & = & \text{to} & = & \text{to} \\ \text{inches} & & \_\_\_\_ & & \_\_\_\_ \end{array}$$

$$\frac{6}{1} = \frac{n}{\_\_\_\_} \qquad n = \_\_\_\_ \times \frac{6}{1} = \_\_\_\_\_\_$$

Answer: _____miles

(J24.) If you made a scale drawing of a boat so that 2 inches = 3 feet, how long will the boat be on your drawing if its real length is 9 feet?

inches ____ what
to = to = to
feet ____ ____

$\frac{2}{3} = \frac{n}{\_}$ n = ____ x ____ = ______

Answer: _____inches

(J25.) If you draw a house to scale so that 2 1/4 inches = 6 feet, then a 20 foot wall would be _?_ inches long on your drawing.

____ ____ what
to = to = to
feet ____ ____

$\frac{\_\_\_\_}{\_\_\_\_} = \frac{n}{\_\_\_\_}$ n = ____ x ____ = ______

Answer: _____inches

Mileage Problems

Example

If you go 100 miles on 6 gallons of gas, how far could you go on 1 gallon? Answer: 16 2/3 miles per gallon

miles		100		what
to	=	to	=	to
gallons		6		1

$\frac{100}{6} = \frac{n}{1}$ $n = \frac{100}{6} \times 1 = 16\ 2/3$

(J26.) How far will your car go on 3 gallons of gas, if you can go 160 miles on 8 gallons?

miles		____		what
to	=	to	=	to
gallons		____		________

n = ____ x ____ = ____ Answer = ____miles

Money Problems

Example

If you pay a tax of $2 for every $10 that you spend, how much tax will you pay on $150? Answer: $30.00

tax		2		what
to	=	to	=	to
cost		10		150

$\frac{2}{10} = \frac{n}{150}$ $n = 150 \times \frac{2}{10} = 30$

(J27.) If the profit on a $50 watch is $2 then the profit on a $75 watch will be $____.

Profit		____		____
to	=	to	=	to
____		____		____

n = ____ x ____ = ____

(J28.) If you are accessed $15 a year for taxes for every $100 worth of property you own, how much are you accessed if you own $6300 in property? Answer: $____

____		____		____
to	=	to	=	to
Property		____		____

n = ____ x ____ = ____